Unit XX **The Simple Pendulum**

The Simple Pendulum

1 Background

We live in a world that is full of movement and motion and one of the purposes of physics is to try and describe and classify these. Of the many movements and motions that we are aware of, one is that of repeated motion. Examples of this are bouncing balls, children on swings, leaves in trees, waves in the sea etc. In looking at these various motions several questions will arise. Why does the bounce of a ball decrease after each bounce? Or why does the child on a swing need to be continually pushed? Why does the leaf in a tree swing back and forth? In order to answer these types of question we need to find out what it is that affects the motion we see, like, what are the forces involved? And are there any energy changes? The Simple Pendulum provides an excellent introduction to the study of Simple Harmonic Motion, Waves and Vibrations and experimental techniques. It can also be used as a tool for explaining conservation of energy, vector diagrams and components, forces and many other physics topics.

2 Study Outline

Topic	Description	Time (mins)	Total
3	Explanation of different motions and demonstrations	10	
4	Pendulum: introduction and discussion	20	
5	Discussion on experiments: what to measure, how to record data and draw graphs	10	
5.1	Experiment/Practical: period vs amplitude	20	
5.2	Experiment /Practical: period vs mass	20	
5.3	Experiment /Practical: period vs length	20	
6	Discussion of results	10	
7	Pendulum: theory, discussion and looking ahead	40	
8	Self-Assessment exercises including extensions and challenges with suggested solutions	180	5 hours

Note that the above times are guidelines. So for example you might well find that 5.1 takes a little longer than 5.2. However there is ample time to complete all the work, and it is possible that some of the challenges are things you carry forward to “tinker” with in the future!

3 Explanation of different Motions

If an object repeats the same pattern of movement over and over, we say that it is ***oscillating*** or ***vibrating***. For example, a child on a playground swing repeats the same movement back and forth many times and is thus ***oscillating***. Also, any event that repeats itself at ***regular*** intervals is said to be ***periodic***.

Some examples of oscillating or vibrating things are:

- Holding a metre rule on the edge of a desk, then pull the end down a little and let go. The end will move up and down or vibrate.
- Your grandfather, rocking back and forth in his rocking chair, could be said to be oscillating.
- The prongs of a tuning fork are vibrating as can be shown by inverting the tips of the fork into a beaker of water.

Glossary

Vibrations and **oscillations***:* an object is *vibrating* or *oscillating* when it moves back and forth, repeating the same pattern of movement more or less regularly.

Periodic: an event is *periodic* if it is repeated at regular intervals. So the **period** would be the time interval between two successive and repeated events

We may be interested in *how often* the pattern of movement is repeated. The number of times the complete movement happens in a given time interval is called the ***frequency.*** The amount of time it takes for one complete movement is called the ***period.*** Physicists measure frequency in *cycles per second*. A frequency of 1 cycle per second is 1 *hertz* (Abbreviation: 1 Hz.)

There is a relationship between the period T and the frequency f. If something has a frequency of 2 Hz it completes two cycles in one second, so the time for a single oscillation, or the period, T, would be 0.5 seconds. Similarly if the frequency is say 10 Hz, then there are 10 cycles each second and the period, T, would be 0.1 seconds, so

$$T = \frac{1}{f} \qquad \text{and} \qquad f = \frac{1}{T}$$

4 The Simple Pendulum

A simple pendulum consists of a small mass (called a *bob*) hanging from a piece of string, Fig. 1, which shows the pendulum in its rest or equilibrium position. The length l of the pendulum is from the point of suspension to the **centre** (or centre of mass) of the bob. So if you just measure the length of string, remember to add the radius of the bob (or the distance to the CoM).

If the pendulum is now pulled aside to position A, Fig. 2, and released, it will swing from A, through E to C and back to A as shown below in Fig. 3, it will then have completed a full *cycle* and the time it took to do this is the pendulum's ***period***, T. The horizontal displacement, d, is known as the ***amplitude*** of the pendulum's swing.

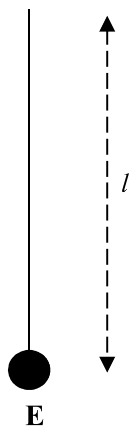


Fig. 1

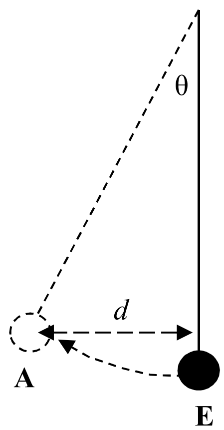


Fig. 2

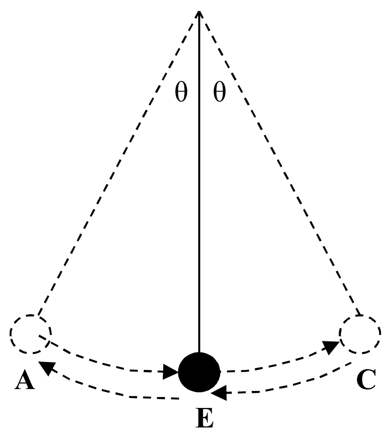


Fig. 3

According to Galileo, the period of a pendulum remains constant as long as the angle between the string and the vertical is *small* and all other properties of the pendulum remain unchanged. But this is not *strictly* true. Because there is air resistance, or friction, the pendulum will slow down and eventually stop. But normally a simple pendulum as shown will swing back and forth, or oscillate, for quite some time.

Setting up a simple pendulum.

For a bob, use one of the suggestions from the needs box, and set it up using one (or any other) of the methods shown in Fig. 4 below. Set the pendulum in motion and watch it oscillate for a few swings. Make sure the string isn't too short or long, usually about 50 cm should do for this exercise.

Needs

For this you will need:
- some string,
- large nut, lead sinker or a set of slotted weights,
- retort stand and clamp, short stick or rod,
- counter weight like a brick

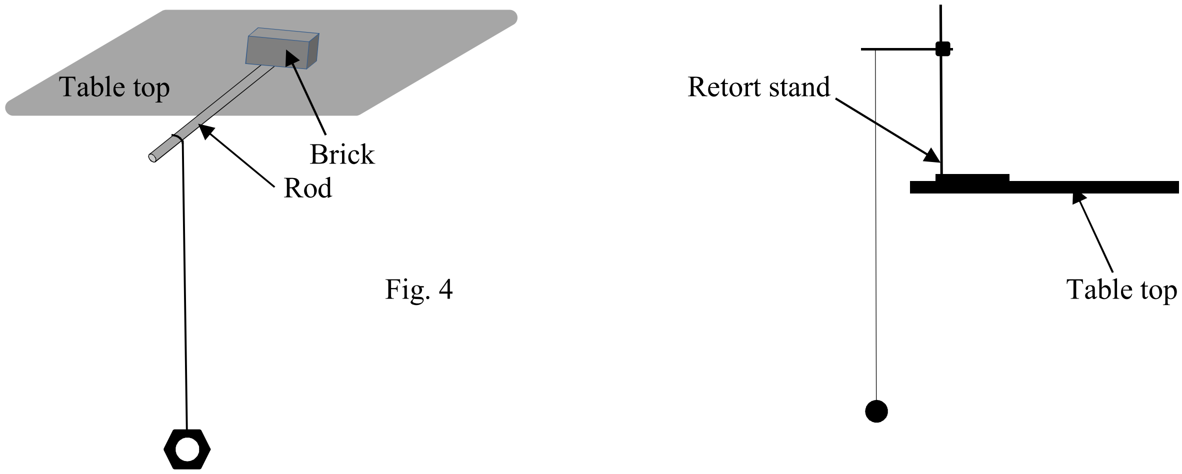


Fig. 4

Repeat using different amplitudes, and then discuss the following questions:

- does the bob move as far to the left as it does to the right?
- does the amplitude noticeably decrease after every swing?
- where does the bob appear to move fastest?
- does this observed speed change with the amplitude you used? And if so, how?
- if you were to try and time the period of the pendulum, would you:
  - release the bob and start your watch at the same time, or
  - would you release the bob, watch it for a swing or two and then start your watch when the bob reaches the end of a swing and then start counting the swings? Remember that as you start your watch count zero, one, two . . .

Recap. At this stage you should know the following:

- what the period of a pendulum is,
- what the amplitude and length of the pendulum is,
- what the relation between frequency and period is, and
- what a complete cycle is.

5 Experiments

The following experiments depend on accurate timing, and it is known that the human reaction time from seeing something to doing something is about 0.1 seconds, see below. So by timing a single swing, the error can be anything from zero to 15%! And you are unlikely to improve on this by repeating this particular method. The best approach for the following experiments is to set the pendulum in motion, watch for a swing or two and then start your timing when the bob reaches position A in Fig. 3, count 10 or 20 swings, record your results in a table as shown below. This will reduce your error to less than 0.2% - a better result! Your table will of course have more rows!

Once you have all the data for an experiment, do whatever calculations are needed, and record these in your table as well. Then use the data you have to plot a graph, and you might well draw more than one graph using the same data in order to achieve the desired outcome of you experiment.

Variable (Units)	**Number of swings**	**Time** (seconds)	**Period *T*** (seconds)

Discuss this experimental procedure with your colleagues and facilitator to make sure you understand what it is that needs doing and how to do it.

> **Reaction time**. To show this, try the following simple exercise with a friend. Your friend holds a standard 30 cm ruler at the 30-cm mark. You place your forefinger and thumb, about 5 cm apart, around the 0-cm mark, so that the ruler is halfway between your thumb and forefinger. Your friend then releases the ruler and you close your fingers as quickly as you can to catch the ruler. If you catch the ruler between the 5 and 10 cm mark, you are doing well: your reaction time is between 0.1 and 0.2 seconds! Work it out using the equations of motion, now or later.

5.1 Investigate the relation between Period and Amplitude

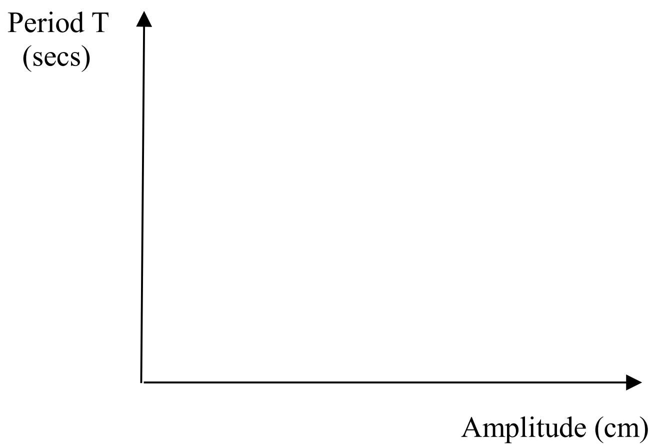
Set up a pendulum as described above, with a length of about 1 m. Remember the length of the pendulum is from the point of suspension to the centre of mass of the bob. Also draw up a results table as described in #5 above, using **Amplitude** as you variable and cm as the unit. Then starting with a small amplitude (see Fig. 2 above), say 5 cm, take the time for about 10 or more complete swings, or oscillations, and record the time on your table. Again, remember to start timing after the pendulum has settled into a regular oscillation.

Now repeat the above, but for an amplitude of say, 10 cm and continue to record results for every 5 cm increase in amplitude. You are free to choose different steps in your investigation.

Once you get to an amplitude of about 40 cm you will probably notice a rapid decrease in the size of the swing or oscillation and by that time should have enough data to plot a graph and reach a conclusion.

Now calculate the period for each amplitude you used and plot the result on a graph using the axes as shown below.

> **Needs**
>
> For this you will need, as before:
> - some string,
> - large nut, lead sinker or a set of slotted weights,
> - retort stand and clamp, short stick or rod,
> - brick counter weight,
>
> But also:
> - a metre rule or tape measure,
> - a stop watch or other timing device: many cell-phones have a stop watch
> - some graph paper



Once you have drawn your graph, compare it with that of some of your colleagues and discuss the results – are they what you expected? What does your result show?

Use your calculator to calculate the angle θ (see Fig 2 and 3 above). Also see for what values of θ, in radians, the following relations are true: $\sin\theta = \tan\theta = \theta$

This approximation is often used in physics

5.2 Investigate the relation between Period and Mass of the bob

Needs

In addition to what you needed for 5.1, you will need:
- a number of large nuts, or
- several lead sinkers of different sizes, or
- best of all, a set of slotted weights,
- a scale to weigh the nuts/sinkers.

Set up your pendulum as in 5.1 above. This time your variable quantity is **Mass** in grams (g). Starting with a relatively small mass, about 50 g set the pendulum going with an amplitude of about 5 cm or so. Once the pendulum has settled, start timing about 10 or more oscillations as before and record your results on the new data table you've drawn up.

Repeat the experiment with a slightly larger mass, say 100 g or so. It is important that you use the same amplitude as you did before, and again record your results.

Repeat this until you have enough data to draw a graph, then calculate the period for each mass. Draw a graph on graph paper and this time use Mass as the variable on the horizontal axis.

And again once you have drawn your graph, compare it with that of some of your colleagues and discuss the results – are they what you expected? What does this result show? What assumption have you made? See discussions in the next section.

5.3 Investigate the relation between Period and the Length of the pendulum

Once again set up your pendulum as in 5.1, however this time make it a little longer, say 1.5 m or thereabouts. Choose a suitable mass, about 100 g or so and again use the same amplitude for each part of this investigation, in which the variable quantity is the length of the pendulum.

Set the pendulum in motion, measure and record the period in your new data table as you did before. Note: when you draw up your table add an extra column for use later.

Then shorten the pendulum a little, by about 5 cm and repeat the experiment until you have enough data: try and get to a length of about 20 cm. Remember the length of the pendulum is from the pivot point to the centre of mass! When you have collected all the data calculate the period for each part of the experiment. In the extra column, headed as $\sqrt{l}$, your table should now look like this, with extra rows of course:

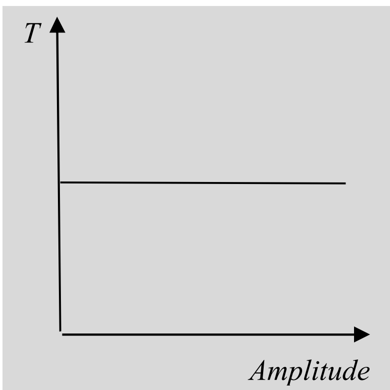
Length (cm)	**Number of Swings**	**Time** (seconds)	**Period *T*** (seconds)	$\sqrt{l}$

Now draw two separate graphs, the first as before with period T on the vertical axis and l on the horizontal axis. On the second change l to $\sqrt{l}$ and use the values from your table to plot the second graph.

And again once you have drawn your graphs, compare them with those of some of your colleagues and discuss the results – are they what you expected? What does this result show? See discussions in the next section.

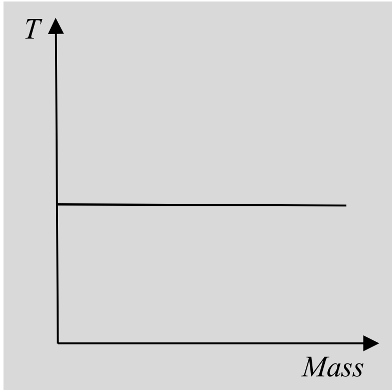
6 Discussion of investigation results

In the first investigation, 5.1 you should have got a graph that looked something like that shown on the right. This indicates that the amplitude (size of swing) does not affect the period of the pendulum.



If the angle $\theta < 10^o$ then sin θ, tan θ and θ are effectively the same, and if $\theta < 5^o$ they are to all intents and purposes, the same.

In the second investigation, 5.2 you should again get a graph that looks that shown on the right. This indicates that the period of the pendulum does not depend on the mass of the bob, or in scientific terms, the period is independent of the mass.



The assumption you probably made was that the length of the Pendulum remained the same, but if you were using slotted weights as your bob, (arguably the best), then it is quite likely that the length measured was from the pivot point to the top of the slotted weights, and so there would be a small error, and as a result your graph may not show that the period was exactly constant.

But the independence of the period on mass is not an unexpected result. Galileo allegedly dropped his two different cannon balls off the Leaning Tower at Pisa to show they fell at the same rate, see Fig. 5.

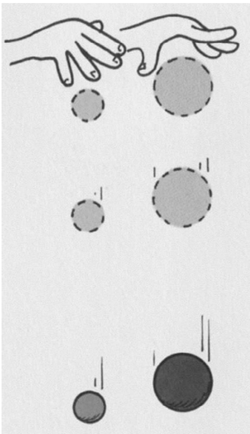


Fig. 5

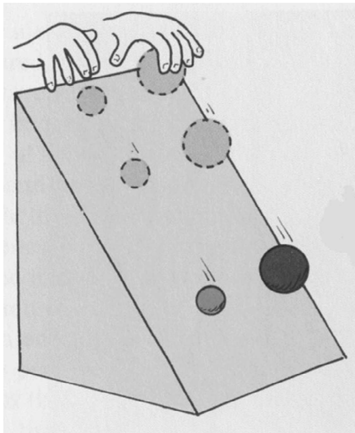


Fig. 6

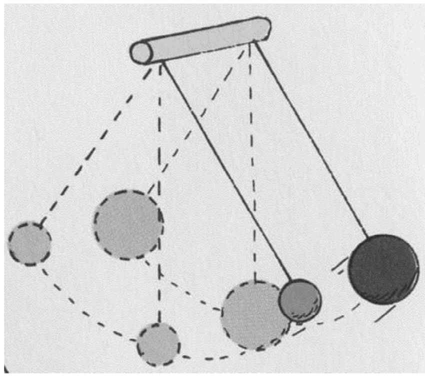


Fig. 7

This idea is then extended by Paul Hewitt in his book "Conceptual Physics" by using the above diagrams to explain why your result is not unexpected. Rolling them down an inclined plane, Fig. 6 is in fact the same as before: you are merely "diluting gravity"! So swinging masses on a string, Fig. 7, would therefore also be the same as they are experiencing the same acceleration, namely *g*.

The final investigation, 5.3, shows that the period is somehow related to the length of the pendulum, as can be seen from your graph. But it is not a straight line, so there is no linear relationship, ie. you cannot say:

$$T \propto l \text{ or } T = kl$$

Now you can try different things to get a straight line and by plotting T against $\sqrt{l}$ you do get a straight line, so it is possible to say that:

$$T \propto \sqrt{l} \quad \text{or} \quad T = k\sqrt{l} \quad \text{or} \quad T^2 = kl$$

If you have plotted T vs $\sqrt{l}$, you can later plot T^2 vs l as well and get the same graph.

Recap: At this stage your investigations have shown that the:

- period of a pendulum does not depend on the mass of the bob or the amplitude of the swing, but
- period does depend on the square root of the length

7 Pendulum: theory, discussion and looking ahead

What are the forces acting on the pendulum? Fig. 8 shows a typical pendulum displaced to one side. If it is now released it will swing back and this is due to the component of the weight of the bob that is *perpendicular* to the string, and thus always acts at a tangent to the path of the bob.

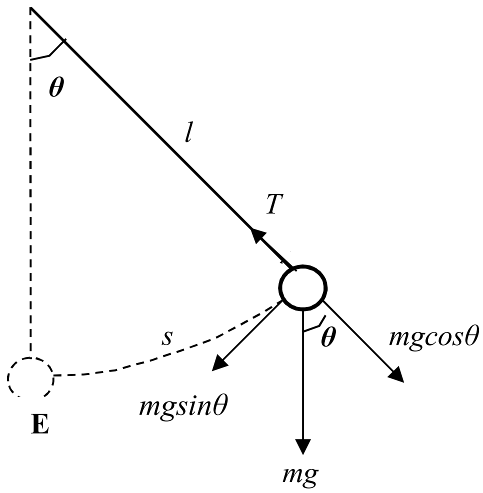


Fig. 8: Forces acting on the bob of a pendulum.

What is also clear is that as θ gets smaller so does the restoring force and when the bob reaches E the restoring force is 0 and the speed of the bob is a maximum. After this the restoring force starts to increase and the bob slows until its speed is 0. As a result the restoring force F is negative and given by:

$$F = -mg \sin\theta$$

The displacement of the bob is the length of the path from where it is at a given instant to its rest position, which is given by:

$$s = l\theta$$

If θ is a ***small angle*** and is measured in *radians*, then:

$$\sin\theta \approx \theta$$

and so: $$F = -\, mg\theta$$

showing that the restoring force is proportional to angle θ, and because *m, l* and *g* are constants

$$F = \frac{-\, mgs}{l} \quad \text{or} \quad F = -\, ks$$

the restoring force is proportional to the (negative) displacement. We'll see the importance of the equation later, and doing some further mathematics it is possible to establish the equation of motion for a simple pendulum as:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

But that mathematics is left for those who are keen! Any of the recommended texts will explain this.

Energy Considerations

When the pendulum bob, mass m is pulled from its equilibrium position R to position A, work is done and the energy of the bob is increased, it now has potential energy, (where g is the acceleration due to gravity)

$$E_P = mgh$$

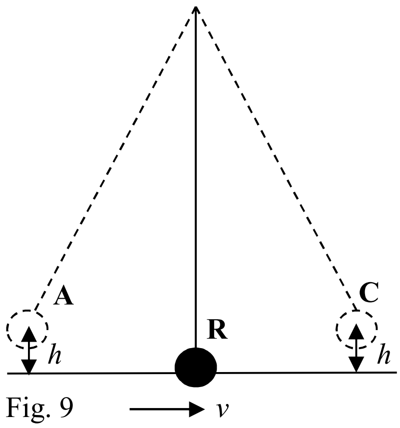


Fig. 9

When the bob is now released, it will swing back to position R, and there the potential energy is 0 and the kinetic energy is

$$E_K = \frac{1}{2} mv^2$$

If we now assume that there is no friction or resistance to motion (there is of course, but it is unnoticeable for a few swings of the pendulum and so the assumption is valid) then the bob will continue to C, height h above the equilibrium position. In this case energy is conserved and we can say:

$$E_P = E_K \quad \text{or} \quad mgh = \frac{1}{2} mv^2$$

So simplifying this we get $\quad v = \sqrt{2gh}$

Thought Experiment

Imagine a hollow pendulum bob filled with ink and an extremely narrow tube allowing a thin stream of ink to come out slowly. If it is then set in motion over a movable sheet of paper, see Fig. 10, it would trace out a sine curve as shown.

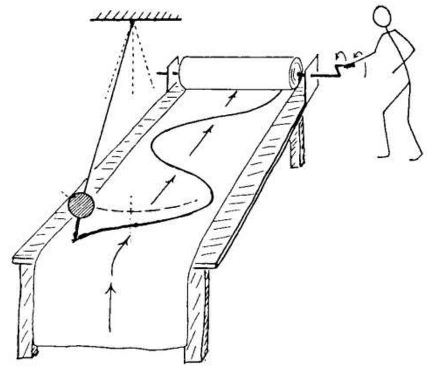


Fig. 10 Making a sine curve with a Pendulum. Image from: E Rogers *Physics for the Inquiring Mind*

Earlier it was shown that a pendulum's motion is such that the restoring force is proportional to the negative displacement and Simple Harmonic Motion, SHM, is defined as:

> *When the restoring force has the form $F = -ks$ the type of motion illustrated in Fig. 10 is designated as Simple Harmonic Motion.*

This of course means that the motion of a pendulum is Simple Harmonic and SHM is often referred to as ***sinusoidal*** as the shape of the graph drawn is that of a sine or cosine function. A study of SHM forms an integral part of wave motion and so this module serves as an introduction to the next module on Waves and Simple Harmonic Motion.

Note: *There are several famous Thought Experiments in physics, and for those interested, simply Google "Thought Experiment"*

8 Self Assessment Exercises

These constitute some problems to help you understand the material a little better and include some guidelines for answering these.

The aim here is to see if you have achieved the learning goals for this module. Do as many as you can in your own time and it is often useful if you speak to colleagues about any difficulties you may experience.

Glossary

Thought Experiments: are devices of the imagination used to investigate the nature of things. These should be distinguished from thinking about experiments, and are often used when it is not actually possible to perform the actual experiment. They are used for diverse reasons in many areas of economics, mathematics, physics, history and philosophy.

The problems are graded as follows:

Exercises – focussed mainly on relatively simple problems to reinforce the material covered in the module,
Extensions – these are more difficult problems aimed at getting you to apply some of what you learnt in this module
Challenges – these are problems, exercises and innovative activities to help you understand some broader aspects of physics and often involve applying many different parts/topics in physics.

Some tips

When solving a problem:

- Read the question carefully and write down the essential aspects of the problem,
- Draw a sketch of the problem and put in the relevant values
- Make sure the units are consistent, don't use lengths in cm and metres, speeds in $m.s^{-1}$ and km/h for example – convert them so that the units match each other,
- If you need to use an equation or formula change the subject of the formula, when necessary, before substituting in the values – and don't put in the units until you give your answer – units often clutter up a relatively simple equation.
- Attempt the problems by yourself. Use the notes and worked examples as a guide. Try to avoid using the guidelines to the solutions, where they are given.
- Sometimes problems appear to have similar answers. These have been included to show how often different wording in the question, actually gives the same or similar answers
- If you have difficulties then consult the available guidelines to the solutions.
- If you still have difficulties in solving one or more problems then consult your tutor

8.1 Exercises

8.1.1 A pendulum makes 90 oscillations in 1minute. What is the pendulums period and frequency?

8.1.2 A pendulum is made from a piece of string 65 cm long with a bob attached to the free end. The bob is pulled to one side through a small angle and released, how long is it before the bob reaches its maximum speed?

8.1.3 Astronauts land on a distant planet and set up a simple pendulum of length 1.2 m. The pendulum swings with SHM and makes 100 complete oscillations in 280 s. What is the acceleration due to gravity in the planet?

8.1.4 The pendulum shown in Fig. 11 has a length of 0.75 m and the bob is released from point A. Find the speed of the bob at point:
- i) C
- ii) B when bob is at 37^{o} to the vertical

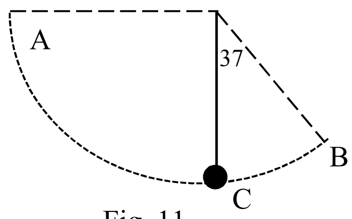


Fig. 11

8.1.5 A simple pendulum has a bob of mass 0.25 kg and a length of 1 m. It is pulled aside so that the string makes an angle of 15^{o} with the vertical, and then released. What is the bob's maximum speed and the maximum restoring force?

8.2 Extensions

8.2.1 The length of a pendulum is tripled, by what factor does the period change?

8.2.2 If the period of a simple pendulum is 2.0 s, what is its length?

8.2.3 A clock, using a pendulum of length 1 m keeps accurate time in a place where g = 9.83 m.s^{-2}. The clock is now moved to a place where g = 9.78 m.s^{-2}. For it to keep accurate time now, what must the length of the pendulum now be?

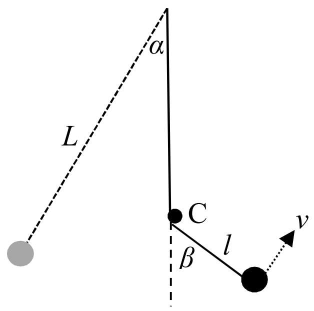
8.2.4 A 0.8 kg bob on a 2 m long cord is pulled sideways until the cord makes an angle of 37° with the vertical and released. Find the work done on the bob and its speed as it passes through the equilibrium position.

8.2.5 A pendulum of length l is pulled aside through an angle θ from its equilibrium position and released. Show that the bob's speed on passing through its equilibrium position has a speed of:

$$v = \sqrt{2gl(1-\cos\theta)} \text{ m.s}^{-1}$$

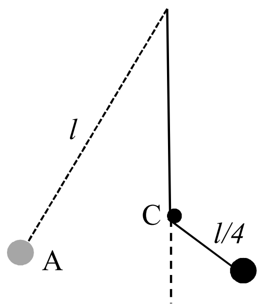
8.3 Challenges

8.3.1 A pendulum of length L is suspended vertically above a peg C. The bob is pulled aside through an angle α as shown and then released. The string supporting the bob strikes peg C and the pendulum then continues as a shortened pendulum of length l. What is the speed v of the short pendulum when it has swung through an angle β ?



8.3.2 A pendulum has a bob of mass M. Show that the tension in the string is T always $\geq Mg$ when the pendulum is swinging normally.

8.3.4 A pendulum has a period T for small oscillations. An obstacle C is placed directly beneath the pivot point so that only the lowest ¼ of the string can follow the bob when it swings to the right of its resting position. The pendulum is pulled to the left to position A and released. How long will it take to return to A*? (The angle between the moving string and the vertical remains fairly small throughout the motion)*



8.3.5 A simple pendulum is hung from the roof of a lift (elevator) at rest. When put in motion the period of the pendulum is T_0. Describe qualitatively what happens to T_0 (if anything) when the lift is:

i. moving upwards at a constant speed
ii. moving downwards at constant speed
iii. moving upwards with constant acceleration
iv. moving downwards with constant acceleration

8.3.4 If a pendulum 1m long were to somehow to be set in oscillation on the surface of a white dwarf star with a mass equal to that of the Sun, but with the radius of the Earth, what would be the pendulum's period? (You may use the following values of the Sun and Earth - $M_\odot = 2.10^{30}$ kg, $R_E = 6.4.10^6$ m and $G = 6.67.10^{-11}$ N.m^{2}kg^{-2})

Glossary

To describe something **qualitatively** means you do so without using the amounts involved, just the reasons or types of things involved, this is as opposed to **quantitatively** when the description involves the amounts or quantities used in the description. Usually the former uses logic and known laws of physics, whilst the latter usually involves doing calculations to find numerical answers/values.

Anwsers and guidelines for solutions to selected problems.

8.1.1 0.67 s and 1.5 Hz

8.1.2 Using the equation for the period of a pendulum:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

And substitute in the value 0.65 m for the l m and 9.81 m.s^{-2} for g, we get

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\times 3.142\times\sqrt{\frac{0.65}{9.81}} = 1.62\,\text{s}$$

But maximum speed occurs after a ¼ period, ie 0.4 s

8.1.3 100 complete swings in 280 s means that the period $T = 2.8$ s. Then using

$$T = 2\pi\sqrt{\frac{l}{g}}$$

and making g the subject of the equation

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{then} \quad T^2 = 4\pi^2\frac{l}{g} \text{ so } \quad g = 4\pi^2\frac{l}{T^2} \quad = \quad 4\times 9.87\times\frac{1.2}{7.84} = 6\,\text{m.s}^{-2}$$

8.1.4 Loss of E_P = gain of E_K, ie mgh = ½ mv^2 so $v = \sqrt{2gh} = 2\times 9.81\times 0.75 = 3.84$ m.s^{-1}

8.1.5 The vertical height h of the bob above its equilibrium position is 1 – 1cos 15 = 0.034 m.

Maximum speed: $v = \sqrt{2gh} = 2\times 9.81\times 0.034 = 0.82$ m.s^{-1}

Maximum restoring force FR = mgsin 15 = 0.25 x 9.81 x 0.26 = 0.63 N

8.2.1 It would be √3 times longer! Take $T = 2\pi\sqrt{\frac{l}{g}}$ and square it, put in the values and take the square roots again,

so $T_0^2 = 4\pi^2\frac{l}{g}$ becomes $T_0 = 2\pi\sqrt{\frac{l}{g}}$

and $T_1^2 = 4\pi^2\frac{3l}{g} = 3\times 4\pi^2\frac{l}{g}$ becomes $T_1 = \sqrt{3}\times 2\pi\sqrt{\frac{l}{g}} = \sqrt{3}T_0$

8.2.3 Using the equation for the motion of a pendulum and changing the subject of the equation to l we get:

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{then} \quad T^2 = 4\pi^2\frac{l}{g} \quad \text{so} \quad l = \frac{gT^2}{4\pi^2} \quad = \frac{9.81\times 4}{4\times 9.87} = 0.99\,\text{m}$$

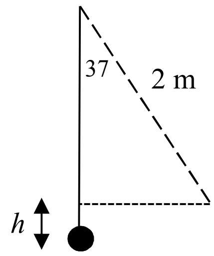
8.2.4 Using the equation of motion for the pendulum and the values given:

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\times 3.142\times\sqrt{\frac{1.0}{9.83}} = 2.00\,\text{s}$$

For the new place we know g and T, so we need to make l the subject of the equation:

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{then} \quad T^2 = 4\pi^2\frac{l}{g} \quad \text{so} \quad l = \frac{gT^2}{4\pi^2} = \frac{9.78\times 4}{4\times 9.87} = 0.99\ \text{m}$$

8.2.5 Remember an angle of 37° is in a 3:4:5 triangle, so the height



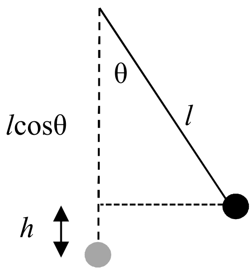
$h = 1 - 1.6 = 0.4$ m

Work done = gain of $E_P = mgh = 0.8 \times 9.81 \times 0.4 = 3.14$ J

Loss of E_P = gain of E_K, ie $mgh = ½\, mv^2$ so

$$v = \sqrt{2gh} = 2\times 9.81\times 0.4 = 2.8\ \text{m.s}^{-2}$$

8.2.5 From the conservation of energy we get:

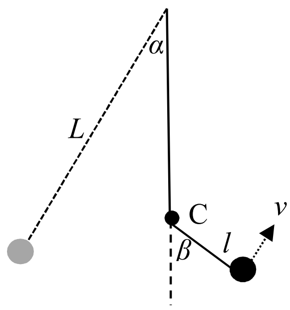


Loss of E_P = gain of E_K, ie $mgh = ½\, mv^2$ ie. $v = \sqrt{2gh}$

Now $h = l - l\cos\theta = l(1 - \cos\theta)$, so

$$v = \sqrt{2gh} = \sqrt{2gl(1-\cos\theta)}\ \text{m.s}^{-1}$$

8.3.1 Using the lowest position of the bob as he reference point for gravitational potential energy, then the potential energy E_P for the bob in the position shown is (see 8.2.4 above)



$$E_P = mgl(1 - \cos\beta)$$

and because it's moving at speed v, it has kinetic energy E_K

$$E_K = ½\, mv^2$$

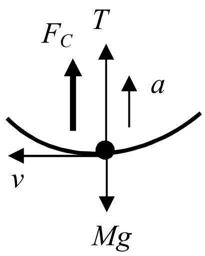
The initial potential energy $U_P = mgL(1 - \cos\alpha)$

Now $U_P = E_P + E_K$ so $E_K = U_P - E_P$ or

$½\, mv^2 = mgL(1 - \cos\alpha) - mgl(1 - \cos\beta)$

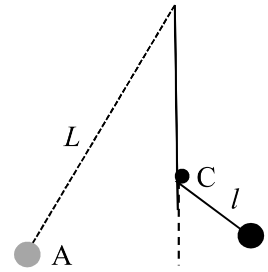
So $v = [gL(1 - \cos\alpha) - gl(1 - \cos\beta)]^{½}$

8.3.2 Obviously when the pendulum is at rest $v = 0$, $T = Mg$ Away from the rest position, $v > 0$ and so the inward acceleration, a > 0 since $a = \frac{v^2}{r}$. The centripetal force



$ma = \frac{mv^2}{r} = F_C$. Now $F_C = T - Mg$ ie $F_C + Mg = T$
Then since $F_C \geq 0$, $T \geq Mg$.

8.3.3 When the bob is to the right of the pivot point C, the peg acts As a pivot point for a pendulum of length $L/4$ (or ¼ $L = l$). When the bob is to the left of the peg the pendulum has a length L. If the pendulum is released from A, it will swing for half a cycle as a pendulum of length L, and half a cycle of length $l = L/4$. The elapsed time is, using equation for pendulum motion



$$\frac{1}{2}\left(2\pi\sqrt{\frac{L}{g}}+2\pi\sqrt{\frac{L}{4g}}\right)$$

and since $T = 2\pi\sqrt{\frac{L}{g}}$ the total time elapsed is $\frac{T}{2}+\frac{T}{4}=\frac{3T}{4}$

8.3.4 For (i) and (ii) the acceleration of the lift is $a = g$ which means that the period $T = T_0$
For (iii) $a > g$, which means that $T < T_0$
For (iv) a < g, which means that $T > T_0$

8.3.5 In order to solve this problem, you need to find the value of g on the white dwarf star, let's call it g_{WD} – and it will have a remarkable value.

Using Newton's Law of Universal gravitation, $F = \frac{GmM}{r^2}$ then using the given values of the Sun and Earth - $M_\odot = 2.10^{30}$ kg, $R_E = 6.4.10^6$ m and $G = 6.67.10^{-11}$ N.m^{2}kg^{-2} we can work out g_{WD}.

$$F = \frac{GmM}{r^2} = mg_{WD} = \frac{GmM_\odot}{r_E^2} \quad \text{so} \quad g_{WD} = \frac{GM_\odot}{r_E^2}$$

$$g_{WD} = \frac{GM_\odot}{r_E^2} = \frac{6.67\times10^{-11}\times2\times10^{30}}{\left(6.4\times10^6\right)^2} = 3.26\times10^6\,m.s^{-2}$$ - quite a value!

Then using $T_{WD} = 2\pi\sqrt{\frac{L}{g}}$ we get $T_{WD} = 2\pi\sqrt{\frac{1}{3.26\times10^6}} = 3.5\times10^{-3}$ or 3.5 ms!

With a gravitational field that strong, strange things happen.
Density is about 6.63 x 10^6 tonnes/m^3 and a 100g bob would weigh 3.26 x 10^5 N!! Imagine the cord to hold this bob!

8.4 Challenging Activities

Possible methods of solving these problems can be given during the course or after from (website address). You will not be formally evaluated for doing these, but rather see them as a challenge that will broaden your knowledge of physics and expose you to some lateral, innovative thinking and showing you how different aspects of physics are inter-related.

You are encouraged to search the internet for ideas, but please don't *cut-and-paste* – you'll get much more satisfaction doing it yourself!

8.4.1 Foucault's Pendulum

A simple pendulum with a very long wire and heavy bob was first made by Leon Foucault, a French physicist to show that the Earth rotated. Whilst this was known, there had never yet been a simple way of showing this. Today many Science Centres and Museums have a Foucault pendulum as they make an impressive showing. Many physics departments also have them, usually in the stairwell! To actually make a Foucault pendulum that keeps going for extended periods of time is not easy, but it is easy to explain to students/people how a Foucault pendulum works.

You can Google "Foucault Pendulum" and you will see many sites – start with the Wikipedia site which has several animations, as do several other sites.

Your challenge is to design and make a desk top model of such a pendulum and use it to demonstrate how the pendulum shows that the Earth rotates.

Discuss with colleagues the motion of the pendulum across the floor beneath it at various parts of the world, such as the poles, Cape Town and the equator, for example.

8.4.2 Bottle Pendulum

Make a simple pendulum of length about 25 cm using the empty bottle as a bob. Make a small hole in the screw cap, thread the string through and tie a couple of knots in it so that you can suspend the bottle so that it hangs down fairly straight.

Weigh the bottle, swing it and try and to time several swings so that you can get the period *T*. Now add about 50 ml (50 ml water = 50 g) of water and repeat, and record all your results as before. Continue to do this until the bottle is full. Now plot a graph of period *T* vs mass *M*. Then:

- comment on the accuracy of your timing,
- explain your graph if it was/wasn't what you expected it to be,
- what do you think would happen if you did the same experiment, but with a string about 10 cm or 2 m long?

Needs

1 Some string.
2 An empty 500 ml plastic water bottles: one of blue ones with a screw top.
3 Water and funnel.
4 A stopwatch or cellphone for timing.
5 A scale that can find the mass to the nearest gram, or a 100/250 ml measuring cylinder

8.4.3 Measuring the maximum speed of a pendulum's bob

The simple pendulum can be used to verify (within simple experimental limits) the Law of Conservation of energy. In order to do this one needs to devise a method of measuring the speed of the pendulum bob as it passes its lowest point, without using electronic measuring devices, but using instead some simple laboratory apparatus and other readily available material.