1. (a) $\angle P = 23°$ [base $\angle's$ of ΔPOQ with $OP = OQ$]

$y = 2(\angle P)$ [central $\angle = 2 \times \angle$ on circumf.]

$= 46°$

$\angle PQR = 90°$ [$\angle$ on a semicircle]

$\therefore \angle OQR = 67°$

$\therefore x = 67°$ [base $\angle's$ of ΔROQ with $OR = OQ$]

(b) $\angle M_2 = 25°$ [base $\angle's$ of ΔKOM with $OK = OM$]

$\angle KOM = 130°$ [$\angle's$ of ΔKOM suppl.]

$y = \frac{1}{2}(\angle KOM)$ [central $\angle = 2 \times \angle$ on circumf.]

$y = 65°$

$x + y = 180°$ [opp $\angle's$ of cyclic quad.]

$\therefore x = 115°$

(c) $x = 30°$ [tan-chord theorem]

$\angle D = x = 30°$ [subt. by BF]

$\angle FBD = 90°$ [$\angle$ on a semicircle]

$\therefore y = 60°$ [sum of $\angle's$ of ΔDFB]

2. $AB \perp OD$ [radius is $\perp$ bisector of chord]

$AD = \frac{1}{2}AB = 6$

$r = x + 2$ [given]

In ΔADO, $r^2 = x^2 + 6^2$ [Pythagoras]

$(x + 2)^2 = x^2 + 36$

$x^2 + 4x + 4 = x^2 + 36$

$\therefore x = 8$ and hence $r = 10$

3. (a) $\angle O_1 = 80°$ [central $\angle = 2 \times \angle$ on circumf.]

(b) $\angle Y_4 = \angle YLO$ [base $\angle's$ of ΔYOL with $OY = OL$]

$\angle Y_4 + \angle YLO + \angle O_1 = 180°$ [$\angle's$ of Δ]

$2\angle Y_4 + 80° = 180°$

$\angle Y_4 = 50°$

(c) $\angle L_7 = \angle O_1 = 80°$ [alt. $\angle's$]

$\angle L_7 = \angle M = 80°$ [base $\angle's$ of ΔMOL with $OM = OL$]

$\angle O_2 + \angle L_7 + \angle M = 180°$ [$\angle's$ of ΔLOM]

$\angle O_2 = 20°$

4. (a) tan-chord theorem.

(b) $\angle D_3 = 90°$ [$\angle$ on a semicircle]

$\angle E = 90° - x$ [$\angle's$ of Δ suppl.]

$\angle A_1 = \angle D_2$ [subt. by chord TC]

$\therefore \angle A_1 = \angle E = 90° - x$ [both $= \angle D_2$]

5. (a) $\angle ACB = 90°$ [$\angle$ on a semicircle]
$\angle ODB = 90°$ [given]
$\therefore \angle ACB = \angle ODB = 90°$
Hence $AC \parallel OE$

(b) $\angle A = x$ [tan-chord theorem]

$\angle BOD = \angle A = x$ [alt. $\angle's$]

(c) $\angle BOE = \angle BCE$ [proved above]
$\therefore OBEC$ is cyclic [$\angle's$ subt. by $BE =$]

(d) $\angle P + \angle PCA + \angle A = 180°$ [$\angle's$ of Δ suppl.]
$\angle P + (90° + x) + x = 180°$
$\angle P = 90° - 2x$

If $\angle ABC = 60° \Longrightarrow x = 30°$
Hence $\angle P = 90° - 2(30°) = 30°$

6. (a) $\angle MPA = 90°$ [tan $\perp$ radius]
Similarly $\angle MRA = 90°$
Hence $APMR$ is cyclic [opp. $\angle's$ suppl.]

(b) $\angle P_2 = \angle R_2$ [base $\angle's$ of ΔMPR with $PM = RM$]
$\angle A_1 = \angle R_2$ [subt. by PM]
Hence $\angle P_2 = \angle A_1$ [both $= \angle R_2$]
MP is tangent to circle APS [tan-chord theorem]

7. (a) $ABDE$ is a cyclic quad. [$\angle E_1 = \angle D_1$ subt by AB both $= 90°$]
$TPCD$ is a cyclic quad. [Ext $\angle ADC =$ opp. int. $\angle TPC$ both $= 90°$]
$CDHE$ is a cyclic quad. [Opp. $\angle's$ HEC and CDH suppl.]

(b) $\angle C_1 = \angle D_5$ [subt. by TP]
$\angle C_1 = \angle B_1$ and $\angle C_1 = \angle A_2$ [tan-chord theorem]

(c) $\angle B_2 = \angle A_2$ [subt. by DE]
$\angle B_1 = \angle A_2$ [subt. by TC]
Hence $\angle B_1 = \angle B_2$ [both $= \angle A_2$]

(d) $\angle B_1 = \angle D_5$ [both $= \angle C_1$]
$\therefore DP$ is tangent to circle BDT [tan-chord theorem]

8. (a) A radius is a perpendicular bisector of a chord.

(b) In ΔRAQ and ΔSAQ
$AR = AS$ [proved above]
$\angle RAQ = \angle SAQ$ [given]
AQ is common
$\Delta RAQ \equiv \Delta SAQ$ $[s, \angle, s]$
$\angle Q_1 = \angle Q_2$ [from congruency]

(c) $\angle RQS = 2t$ [given]

(i) $\angle RTS = \angle RQS = 2t$ [subt. by RS]

(ii) $\angle RSQ = \angle SRQ$ [from congruency]
$\angle RSQ + \angle SRQ + \angle RQS = 180°$ [$\angle's$ of Δ suppl.]
$2\angle RSQ + 2t = 180°$
$\angle RSQ = 90° - t$
OR
$\angle RSQ + \angle SQA + \angle SAQ = 180°$ [$\angle's$ of Δ suppl.]
$\angle RSQ + t + 90° = 180°$
$\angle RSQ = 90° - t$

(iii) $\angle POR = 2\angle Q_1 = 2t$ [central $\angle = 2 \times \angle$ on circumf.]
$\therefore \angle ROA = 180° - 2t$ [adj. $\angle's$ on a straight line]

(d) $\angle POR = \angle RTB$ [both $= 2t$]
Hence $ROBT$ is cyclic. [ext. $\angle =$ opp. int. $\angle$]

9. 9.1. $\frac{CD}{AC} = \frac{BE}{AB}$ [line drawn ∥ to one side of Δ]
$\frac{CD}{3} = \frac{3}{1} \Longrightarrow CD = 9$ units.

9.2. $x + 3 = 9 \Longrightarrow x = 6$ units

9.3. In ΔABC and ΔAED

$\angle A$ is common

$\angle ABC = \angle AED$ and $\angle ACB = \angle ADE$ [corr. $\angle's$]

Hence $\Delta ABC|||\Delta AED$

$\therefore \frac{BC}{ED} = \frac{AB}{AE}$ [from similarity]

$\frac{BC}{9} = \frac{1}{4} \Longrightarrow BC = 2{,}25$ units

9.4. $\frac{\text{area of } \Delta ABC}{\text{area of } \Delta GFD} = \frac{\frac{1}{2}AC.BC.Sin\angle ACB}{\frac{1}{2}GD.FD.Sin\angle GDF}$

$= \frac{AC.BC}{GD.FD}$ [$\angle ACB = \angle GDF$]

$= \frac{3(2{,}25)}{9(3)} = 0{,}25$

10. 10.1. $\frac{MS}{SR} = \frac{MN}{NP}$ [line drawn ∥ to one side of Δ]

$\frac{12}{SR} = \frac{2r}{\frac{2r}{3}} \Longrightarrow SR = 4$ units

10.2 In ΔMSN and ΔMRP

$\angle M$ is common

$\angle MSN = \angle MRP$ and $\angle MNS = \angle MPR$ [corr. $\angle's$]

Hence $\Delta MSN|||\Delta MRP$

$\therefore \frac{SN}{RP} = \frac{MN}{MP}$ [from similarity]

$\frac{SN}{12} = \frac{2r}{\frac{8r}{3}} \Longrightarrow SN = 9$ units

10.3. In $\Delta MSN, \angle MSN = 90°$ [∠ on a semicircle]

$MN^2 = MS^2 + SN^2$ [Pythagoras]

$(2r)^2 = 12^2 + 9^2$

$4r^2 = 144 + 81 \Longrightarrow r = 7{,}5$ units

11. 11.1. $BC \parallel AD$ [opp. sides of $\parallel^m$]

$\therefore FK \parallel AD$ [both ∥ to BC]

$CF = FA$ [diagonals of a $\parallel^m$bisect]

$\frac{CK}{KP} = \frac{CF}{FA}$ [line drawn ∥ to one side of ΔCAP]

$\frac{CK}{KP} = 1$ [since $CF = FA$]

Hence $CK = KP$

11.2. $\frac{BS}{BA} = \frac{5}{2}$ [given]

$\frac{BS-BA}{BA} = \frac{5-2}{2} \Longrightarrow \frac{SA}{AB} = \frac{3}{2}$

$\frac{SP}{PC} = \frac{SA}{AB}$ [line drawn ∥ to one side of ΔBCS]

$\frac{SP}{PC} = \frac{3}{2} \Longrightarrow \frac{PC}{SP} = \frac{2}{3} \Longrightarrow \frac{2KC}{SP} = \frac{2}{3} \Longrightarrow \frac{KC}{SP} = \frac{1}{3}$

Hence $KC: SP = 1: 3$

11.3. In ΔSAP and ΔSBC
$\angle S$ is common
$\angle SAP = \angle SBC$ and $\angle SPA = \angle SCB$ [corr. $\angle's$]
Hence $\Delta SAP\,|||\Delta SBC$
$\therefore \frac{AP}{BC} = \frac{SP}{SC}$ [from similarity]
$\frac{AP}{15} = \frac{3}{5} \Longrightarrow AP = 9$ units

12.

12.1.1. $\angle ADB = 90°$ [$\angle$ on a semicircle]
$OS \parallel AC$ [co-int. $\angle's$ suppl.]

12.1.2. $\angle ADB = 90°$ [$\angle$ on a semicircle]
$\angle ABC = 90°$ [tan ⊥ radius]
In ΔABC and ΔADB
$\angle A$ is common
$\angle ABC = \angle ADB = 90°$
Hence $\Delta ABC|||\Delta ADB$ [$\angle, \angle, \angle$]
Similarly $\Delta ABC|||\Delta BDC$
Hence $\Delta ABC|||\Delta ADB|||\Delta BDC$

12.1.3. From $\Delta ABC|||\Delta ADB$
$\frac{AB}{AD} = \frac{AC}{AB}$ [from similarity]
$\therefore AB^2 = AD.AC$

12.2. $\frac{BS}{SC} = \frac{BO}{OA}$ [$OS \parallel AC$ in ΔABC]

$\therefore BS = SC$ [since $BO = OA$, radii]

$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta OSB} = \frac{\frac{1}{2}AB.BC}{\frac{1}{2}OB.BS} = \frac{2.OB.2BS}{OB.BS} = 4$

13.1 $\angle B_2 = \angle D_2$ [base $\angle's$ of ΔBMD with $MB = MD$]
$\angle B_1 = \angle D_1$ [given]
Hence $\angle ABD = \angle MDC$
$\angle CMB = \angle BAD$ [$2\angle BAD = \angle BMD$ and $\angle M_1 = \angle M_2$]
$\angle MCD = \angle ADB$ [sum $\angle's$ of Δ]

$\therefore \Delta ABD ||| \Delta MDC$

$\frac{AB}{MD} = \frac{BD}{DC}$ [from similarity]

13.2. $BJ = JD$ [$\Delta BMJ \equiv \Delta DMJ$]

$\frac{AB}{MD} = \frac{BD}{DC}$ [proved above]

$AB = \frac{BD.MD}{DC}$

$= \frac{2.JD.MB}{DC}$ [$2JD = BD$ and $MB = MD$]