FACULTY OF SCIENCE

SOWETO SCIENCE CENTRE (SSC)

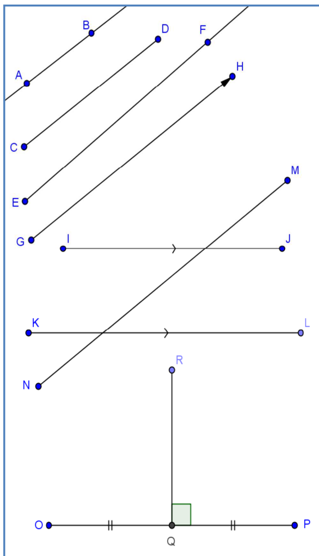
Teachers Workshop

Subject: Mathematics (Euclidean Geometry)

Facilitator: Namadzavho B Kone

The line

❖ Know your lines



AB is a **line** through two points.

CD is a **line segment** joining two points.

EF is a **ray** from E passing through F.

GH is a **vector** from G to H.

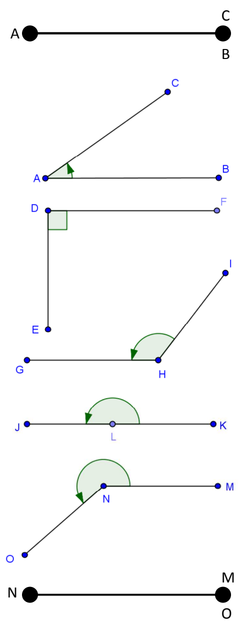
IJ and KL are **parallel** (i.e. $\overline{IJ} \parallel \overline{KL}$) with a **transversal** MN.

RQ is a **perpendicular bisector** of OP with point Q the **midpoint** of OP.

The angle

Definition: A measure of how far a line opens from another line from a fixed point. The unit of measurement is degree (°)

❖ Know your angle type



If point C of the second diagram is moved clock-wise to coincide with point B, then the size of the angle BÂC is **0°.**

BÂC is an **acute angle**.

EDF is a **right angle (90°)**. When two or more angles add up to 90°, they are called **complementary angles.**

IĤG is an **obtuse angle**.

The size of KL̂J on a straight line JK is **180°.** When two or more angles have a sum of 180°, they are called **supplementary angles.**

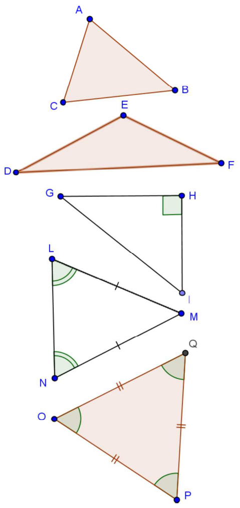
MN̂O is a **reflex angle**.

If O of the previous diagram goes on to coincide with M, then the angle so formed is a **revolution** and it is **360°.**

The triangle

Definition: A closed figure with three linear sides and three angles.

❖ Know your triangle type



ΔABC is an **<u>acute-angled</u>** triangle.

ΔDEF is an **<u>obtuse-angled</u>** triangle.

ΔGHI is a **<u>right-angled</u>** triangle.

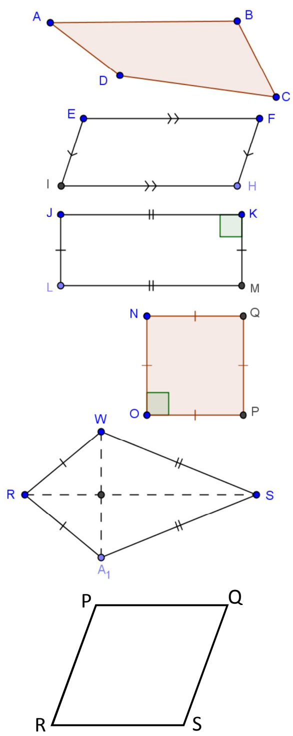
ΔLMN is an **<u>isosceles</u>** triangle.

ΔOQP is an **<u>equilateral (or equiangular)</u>** triangle.

<u>The Quadrilateral</u>

Definition: A closed figure with four linear sides and four inside angles.

❖ Know your quadrilateral type



With PQ = QS = RS = PR

ABCD is just a quadrilateral. All the forthcoming quadrilaterals have special characteristics and hence special names:

EFHI is a **<u>parallelogram.</u>**

JKML is a **<u>rectangle</u>**.

NQPO is a **<u>square.</u>**

$RWSA_1$ is a **<u>kite.</u>**

PQSR is a **<u>rhombus.</u>**

Characteristics of basic geometric shapes

SHAPE	CHARACTERISTIC(S)
Acute Angled Triangle	Each of the three interior angles is less than 90°.
Obtuse Angled Triangle	One interior angle is more than 90°.
Right Angled Triangle	One interior angle is 90°.
Isosceles Triangle	Two sides are equal or base angles are equal.
Equilateral/Equiangular Triangle	Three sides are equal or three angles are equal.
Parallelogram	❖ Opposite sides are equal and parallel. ❖ Opposite angles are equal. ❖ Diagonals bisect each other.
Rectangle	❖ Opposite sides are equal and parallel. ❖ Each angle is 90°. ❖ Diagonals bisect each other and are equal.
Square	❖ All sides are equal and opposite sides are parallel. ❖ Each angle is 90°. ❖ Diagonals bisect each other at right angles and are equal.
Kite	❖ Two pairs of adjacent sides are equal. ❖ One pair of opposite angles is equal. ❖ The smaller diagonal is bisected by the other one at right angles.
Rhombus	❖ All sides are equal and opposite sides are parallel. ❖ Opposite angles are equal. ❖ Diagonals bisect each other at right angles.

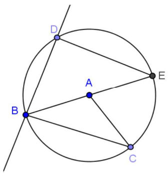
NOTES:

- An isosceles triangle can be acute angled, obtuse angled or right angled.
- An equilateral triangle is always equiangular (each angle size 60°).
- All characteristics of a parallelogram are true for a rectangle, square and rhombus, but the latter three have their own defining (distinguishing) characteristics that make them unique shapes.
- A square is a special type of a rectangle, and a rhombus is a special type of a parallelogram.

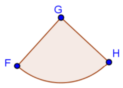
The Circle

➢ Know your circle

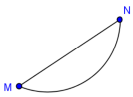
A circle with centre A has the following features:



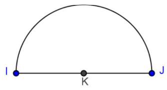
- Line segment BAE is called a **diameter**.
- Each of line segments AB, AE or AC is a **radius**.
- The line through B and D is called a **secant**.
- Each of line segments BD, BC or DE is a **chord**.
- The curved part CE is called an **arc**.



A piece cut out of a circle with centre G is called a **sector**.



A piece cut out of a circle with chord MN is called a **segment**.



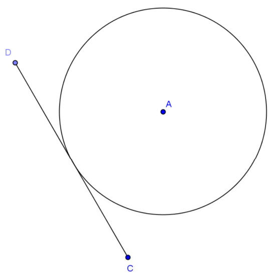
A piece cut out of a circle with diameter IKJ is called a **semi-circle**.

NOTES:

➢ The sector FGH above is actually a **minor sector** with its remaining bigger part to complete the circle called a **major sector**.

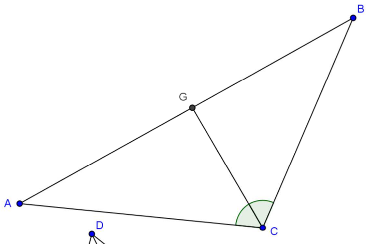
➢ Similarly, the segment MN above is actually a **minor segment** with its remaining bigger part called a **major segment**.

➢ A semi-circle with its diameter has all the qualities of either a segment or a sector!

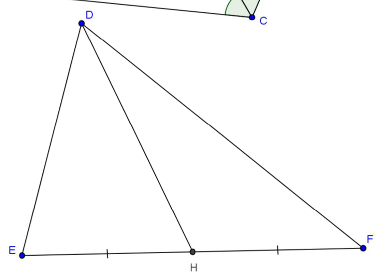


A line segment that touches the circle at only one point (like DC in the accompanying figure) is called a **tangent** to the

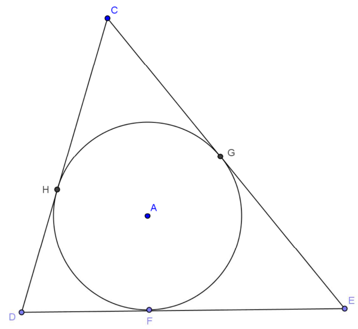
.



A line segment GC that bisects angle ACB of ΔABC is called **<u>an angle bisector.</u>**

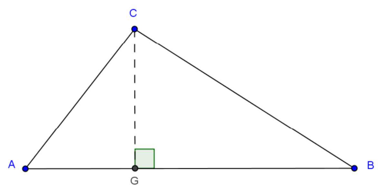


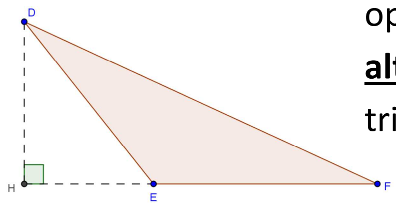
A line segment DH, joining the midpoint of side EF of ΔDEF and the opposite vertex D is called **<u>a median.</u>**



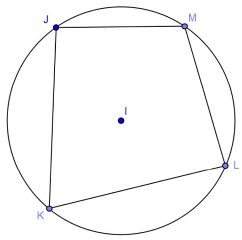
A triangle with its three sides as the tangents of a circle is called **<u>a circumscribed triangle.</u>** The circle is called an **<u>inscribed circle.</u>**

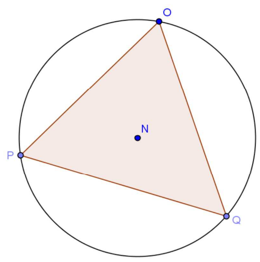
Procedure to draw: Draw angle bisectors on each angle of a triangle and they will meet at one point, then that point is the centre of the circle which will touch each of the three sides of the triangle once.





A line drawn from one vertex perpendicular to the opposite side is called **<u>an altitude or height</u>** of a triangle.





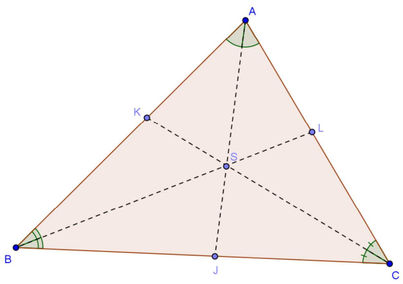
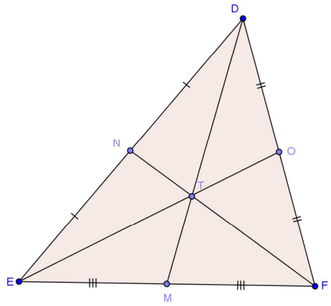
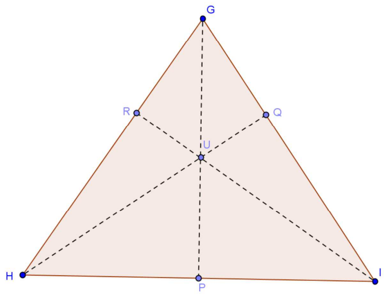
A circle drawn around a polygon with all vertices of the polygon on the circle is called **<u>a circumscribed circle.</u>**

Procedure to draw: After drawing the polygon, draw perpendicular bisectors of sides of the polygon, they meet on one point and that point is the centre of a circumscribed circle.

A quadrilateral JMLK with all its vertices on a circle is called **<u>a cyclic quadrilateral.</u>**

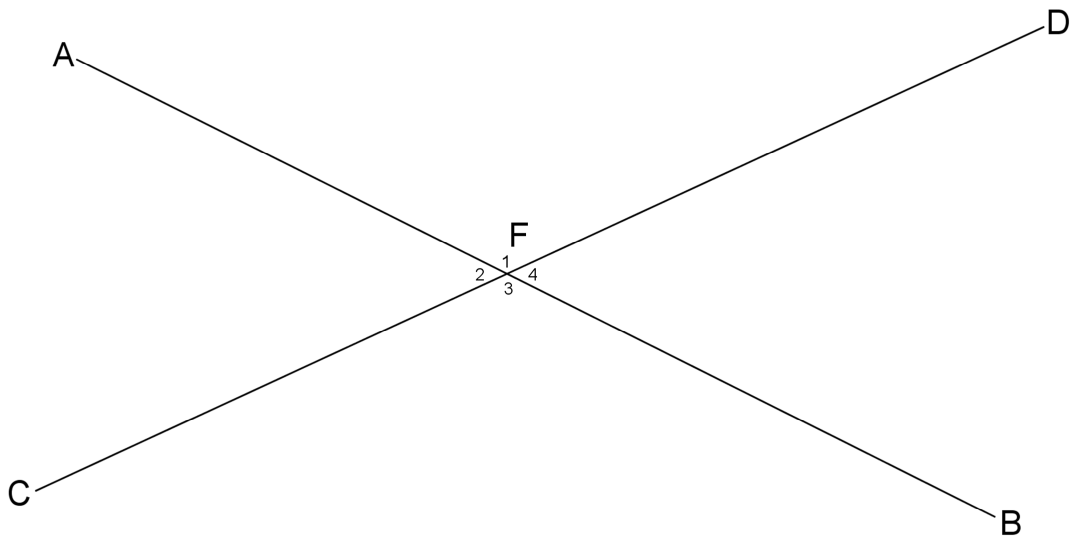
A triangle POQ with all its vertices on a circle is called **<u>an inscribed triangle.</u>**

The centres I and N of the circles above are called **<u>circumcenters</u>**

		
Angle bisectors of angles of a triangle meet at the same point S, and the point is called **<u>an incenter</u>**	Side bisectors of sides of a triangle meet at the same point T, and the point is called **<u>a centroid</u>**	Altitudes of a triangle meet at the same point U, and the point is called **<u>an orthocenter</u>**

Use a tracing paper to measure the specified angles and write down your conclusion from each instruction on the space provided. For measuring length, a 30cm ruler may be used.

Intersecting Lines



Instructions	Conclusion
1. Trace the size of $\hat{F}_1$ and search for any other angle in the diagram equal to it. Repeat #1 for $\hat{F}_2$. 2. Fit the traced $\hat{F}_1$ back to the diagram and shade the size of $\hat{F}_4$; then slide the tracing paper along line segment AB. What size do both $\hat{F}_1$ and $\hat{F}_4$ form? 3. What can you say about $\hat{F}_1 + \hat{F}_4 + \hat{F}_3 + \hat{F}_2$?	**1.** $\hat{F}_1 = \hat{F}_3$ $\hat{F}_2 = \hat{F}_4$ **2.** $\hat{F}_1 + \hat{F}_4 = 180^\circ$ 3. $\hat{F}_1 + \hat{F}_4 + \hat{F}_3 + \hat{F}_2$ $= 360^\circ$

NOTES:

- When two lines intersect, they form a letter X and angles $\hat{F}_1$ and $\hat{F}_3$ OR $\hat{F}_4$ and $\hat{F}_2$ so formed are called **vertical opposite angles.**
- $\hat{F}_1$ and $\hat{F}_2$ OR $\hat{F}_3$ and $\hat{F}_2$ OR $\hat{F}_4$ and $\hat{F}_3$ OR $\hat{F}_1$ and $\hat{F}_4$ lie next to each on a straight line and hence they are called **adjacent angles** on a straight line.

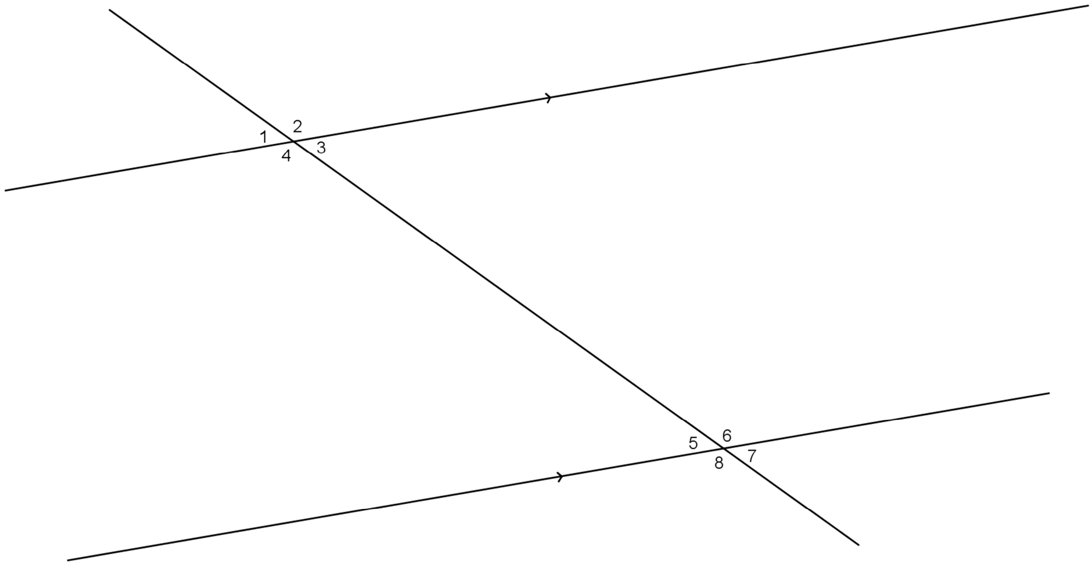
Summarize your conclusions in words:

Theorem 1: **If two lines intersect, then vertical opposite angles are equal.**

Theorem 2: **The sum of adjacent angles on a straight line is 180°. OR Adjacent angles on a straight line are supplementary.**

Theorem 3: **The sum of angles around a fixed point is 360°.**

Parallel lines



Instructions	Conclusion
4. Trace the size of $\angle_1$ and search for all other angles in the diagram equal to it. Repeat #4 for $\angle_8$.	**4.** $\angle_1 = \angle_3 = \angle_5 = \angle_7$ $\angle_8 = \angle_6 = \angle_4 = \angle_2$
5. Shade $\angle_4$; and adjacent to $\angle_4$ on the tracing paper, shade $\angle_5$. Repeat #5 for $\angle_3$ and $\angle_6$. What size does each combination form?	**5.** $\angle_4 + \angle_5 = 180^\circ$ $\angle_3 + \angle_6 = 180^\circ$

NOTES:

- Angles formed with a shape of an F (which can be of any shape amongst others or or or) like $\angle_3$ and $\angle_7$ or $\angle_8$ and $\angle_4$ or $\angle_2$ and $\angle_6$ or $\angle_1$ and $\angle_5$ are called **corresponding angles**.
- Angles formed with a letter Z (which can be of any shape amongst others or or or) like $\angle_3$ and $\angle_5$ or $\angle_6$ and $\angle_4$ are called **alternate angles**.
- Angles formed with a shape of a letter C (which can be of any shape such as or) like $\angle_4$ and $\angle_5$ or $\angle_3$ and $\angle_6$ are called **co-interior angles**.

Summarize your conclusions in words:

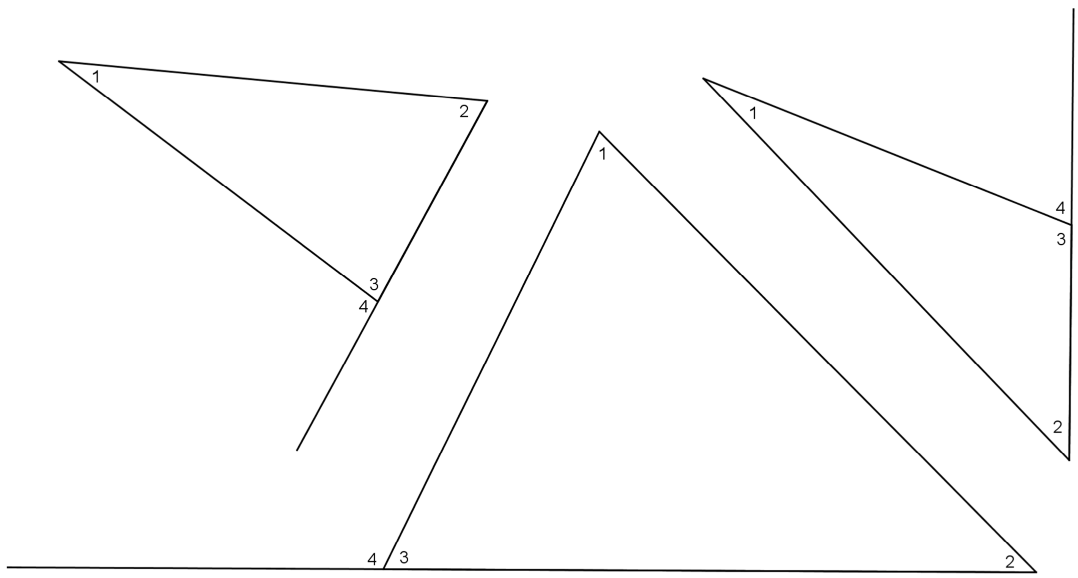
Theorem 4: **If two lines are parallel and cut by a transversal (name two results),**

4.1 alternate angles are equal

4.2 corresponding angles are equal.

Theorem 5: **If two lines are parallel and cut by a transversal, co-interior angles are supplementary.**

Triangle Theorems



NB: Work in one triangle at a time

Instructions	Conclusion
6. Trace the size of $\angle_1$ and adjacent to it on the tracing paper, trace the size of $\angle_2$. Look for an angle equal to the combination of $\angle_1$ and $\angle_2$.	**6.** $\angle_1 + \angle_2 = \angle_4$
7. Take the combination in #6 above and shade $\angle_3$ adjacent to them.	**7.** $\angle_1 + \angle_2 + \angle_3 = 180^\circ$

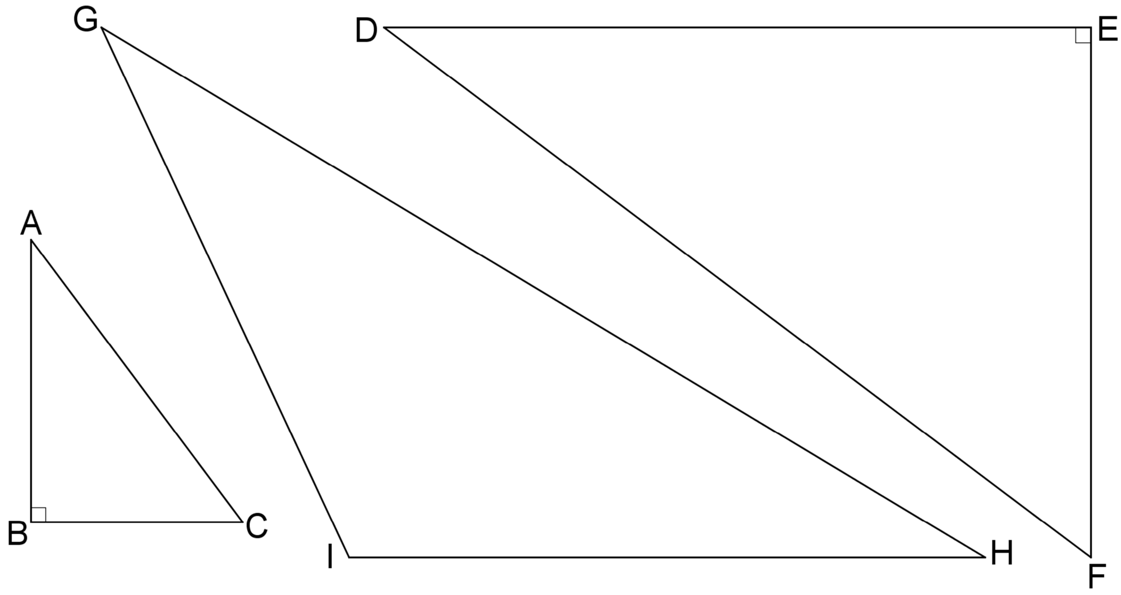
NOTES:

- If a line segment from a triangle is extended outside the triangle, it forms an angle (like $\angle_4$) with one side of the triangle outside the triangle; the angle so formed is aptly named the **exterior angle** of a triangle.

Summarize your conclusions in words:

Theorem 6: **The exterior angle of a triangle is equal to the sum of opposite interior angles.**

Theorem 7: **The sum of angles of a triangle is 180°. OR Angles of a triangle are supplementary.**



<table>
<tr><th>Instructions</th><th>Conclusion</th></tr>
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8. Measure the lengths of sides of each triangle and fill in the table below:

Side	Length	(Length)^2	Value
AB		AB^2	
BC		BC^2	
AC		AC^2	
DE		DE^2	
EF		EF^2	
DF		DF^2	
GH		GH^2	
GI		GI^2	
HI		HI^2	

In each triangle, check the relationship of the squares of the sides.

</td><td>

8.

In ΔABC, $\boldsymbol{AB^2 + BC^2 = AC^2}$

In ΔDEF, $\boldsymbol{DE^2 + EF^2 = DF^2}$

In ΔGHI, there is no relationship amongst the squares of the sides. i.e. $\boldsymbol{HI^2 + GI^2 \neq GH^2}$

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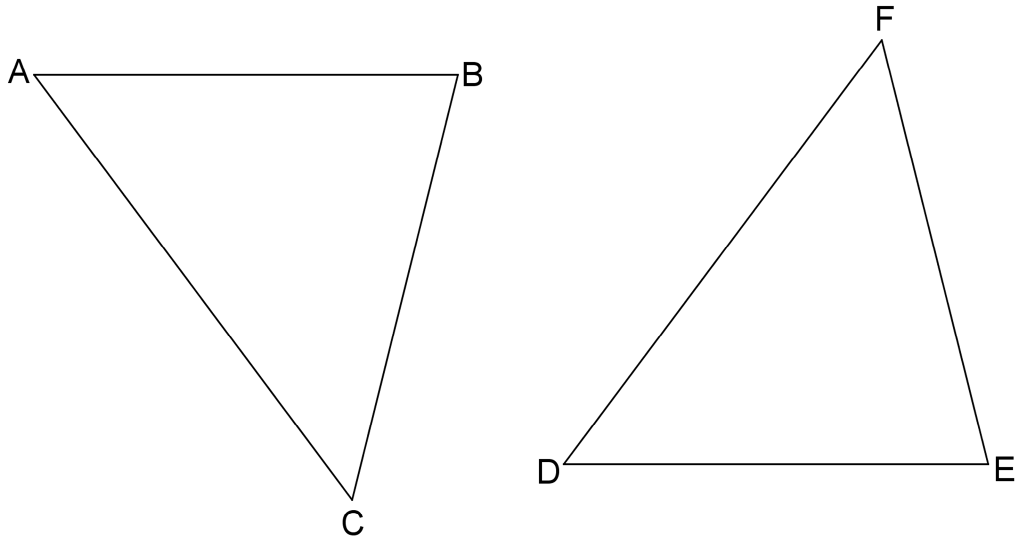
NOTES:

- The two triangles, ΔABC and ΔDEF, each have an angle equal to 90°.
- A side opposite the right angle in a triangle, such as AC and DF is called the **hypotenuse**.

Summarize your conclusion in words

Theorem 8: **In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.**



Instructions	Conclusion
9. Trace ΔABC and try to match it with ΔDEF and write down all things (if any) that are equal from the two triangles.	**9. $\hat{A} = \hat{F}$; $\hat{B} = \hat{E}$; $\hat{C} = \hat{D}$ and AB = EF; AC = DF ; BC = ED**

NOTES:

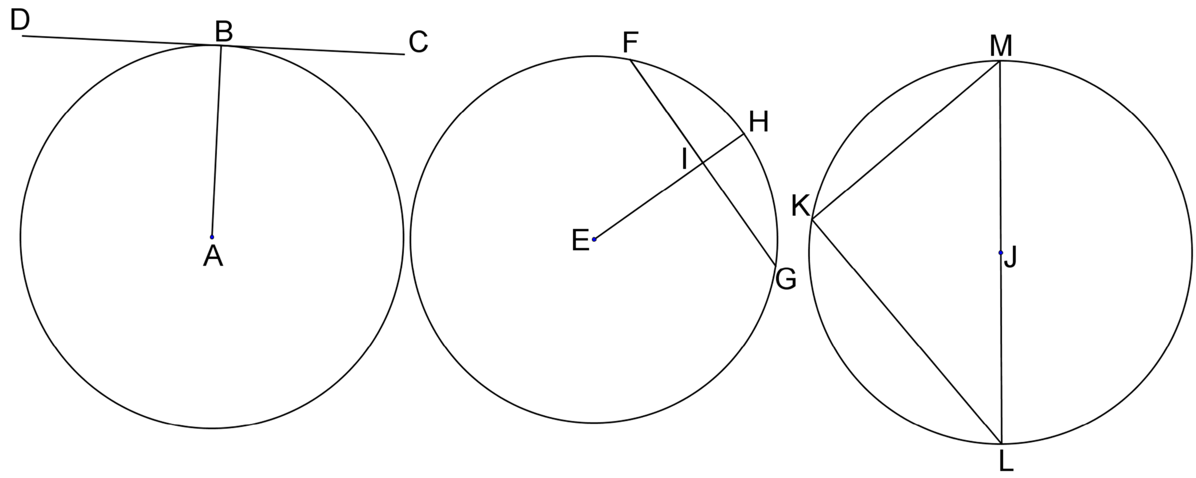
- ΔABC fits exactly onto ΔDEF, i.e. the two triangles are equal in all respects; this phenomenon is called **congruency**. **We say ΔABC is congruent to ΔDEF or can be summarized as** ΔABC ≡ ΔDEF**.**
- $B\hat{A}C$ made by line segments AB and AC is an **included angle** between the two line segments. The same can be said about $E\hat{D}F$ and line segments DE and DF etc.
- Line segment BC is part of both angles CBA and BCA and hence it is the two angles' **corresponding side**. The same applies to EF and $D\hat{E}F$ and $E\hat{F}D$ etc.

THE FOLLOWING ARE CONDITIONS FOR TWO TRIANGLES TO BE CONGRUENT. NB: ANY ONE CONDITION IS SUFFICIENT FOR TWO TRIANGLES TO BE CONGRUENT.

Theorem 9: Two triangles are said to be congruent if (name four conditions):

9.1 three sides of one triangle are respectively equal to three sides of the other triangle,
9.2 two sides and an included angle in one triangle are respectively equal to two sides and an included angle in the other triangle,
9.3 two angles and a corresponding side in one triangle are respectively equal to two angles and a corresponding side in the other triangle and
9.4 **the hypotenuse and one side of one right-angled triangle are respectively equal the hypotenuse and one side**

The Circle Theorems



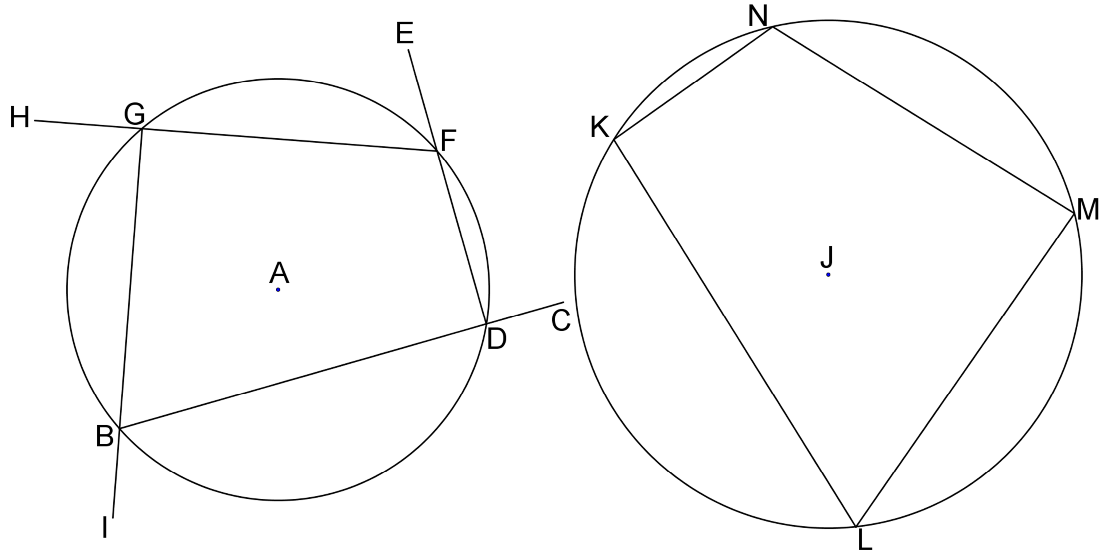
In the accompanying diagrams, A is the centre of the circle and DBC a tangent to the circle while in the other circle with centre E, the radius EH intersects chord FG at I. MJL is the diameter of the circle with centre J and K is a point on the circumference of the circle.

Instructions	Conclusion
10. Measure and compare the sizes of angles ABC and ABD. 11. Measure and compare the sizes of angles EIF and EIG. Measure and compare the lengths of segments FI and IG. 12. Measure and estimate the size of angle MKL.	

Corollary 10:

Theorem 11:

Theorem 12:



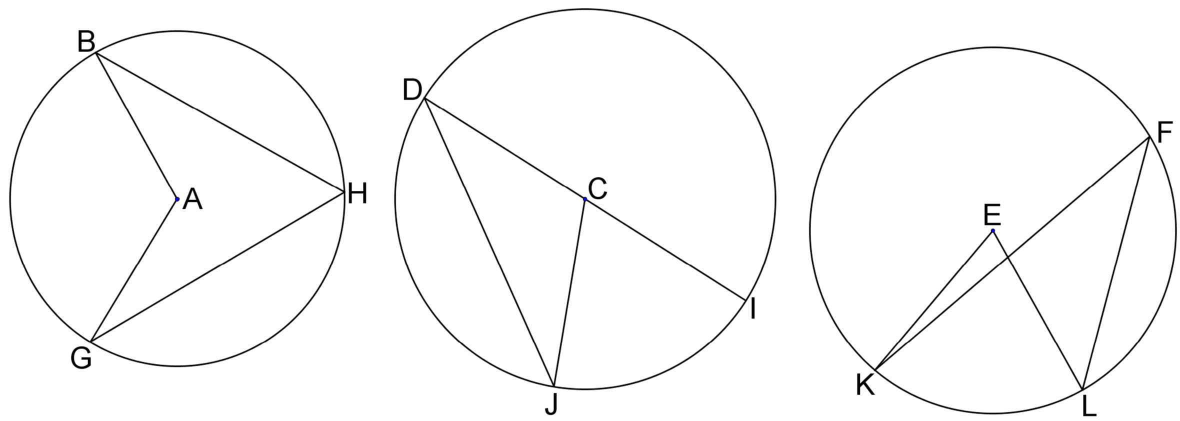
In the accompanying figure, A and J are centres of the respective circles and points B, D, F, G, K, L M and N are points on the circumference of the circles.

Instructions	Conclusion
13. Shade the size of angle IBD and search for an angle equal to it in the same diagram. # Repeat that for angles CDF, EFG and HGB. 14. Shade the size of angle MKL and adjacent to it shade the size angle MNL. # Repeat that for angles KLN and KMN.	

NOTES: An angle formed by extending a side of a cyclic quad with the adjoining chord is called **<u>an external angle</u>** of a cyclic quad.

<u>Theorem 13:</u>

<u>Theorem 14:</u>

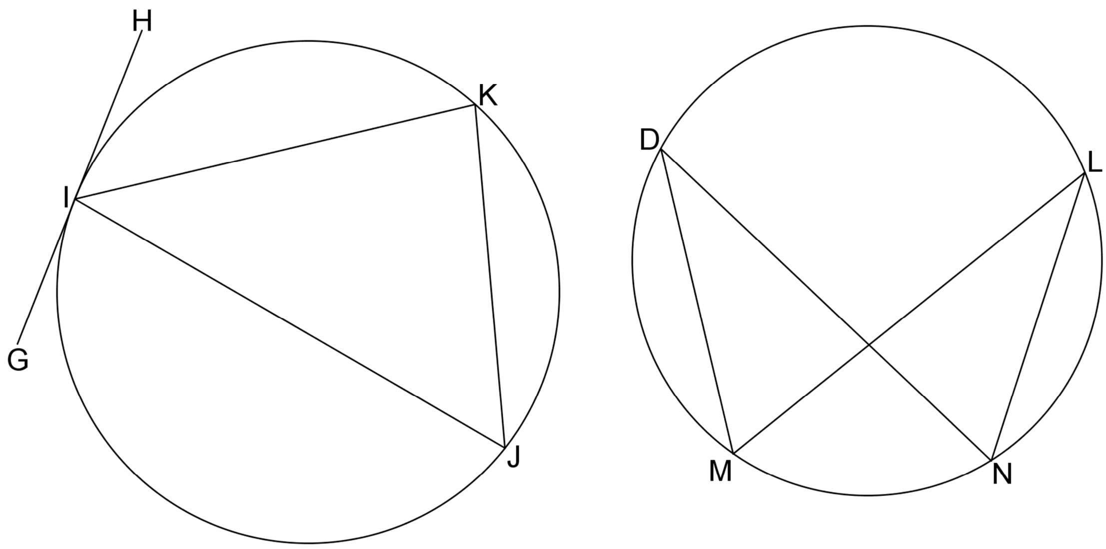


In the above circles, points A, C and E are centres of respective circles and points B, G and H; D, J and I as well as F, K and L are on the circumference of the circles respectively.

Instructions	Conclusion
15. Shade the size of angle BHG and adjacent to it, again shade the same angle BHG resulting in a shade representing double angle BHG then search in the same circle an angle equal to that shade. # Repeat that for angles JDI and KFL in the other circles respectively.	

NOTES: What the three diagrams have in common is that the angle at the centre and the angle on the circumference are **<u>subtended</u>** by the same arc.

<u>Theorem 15:</u>

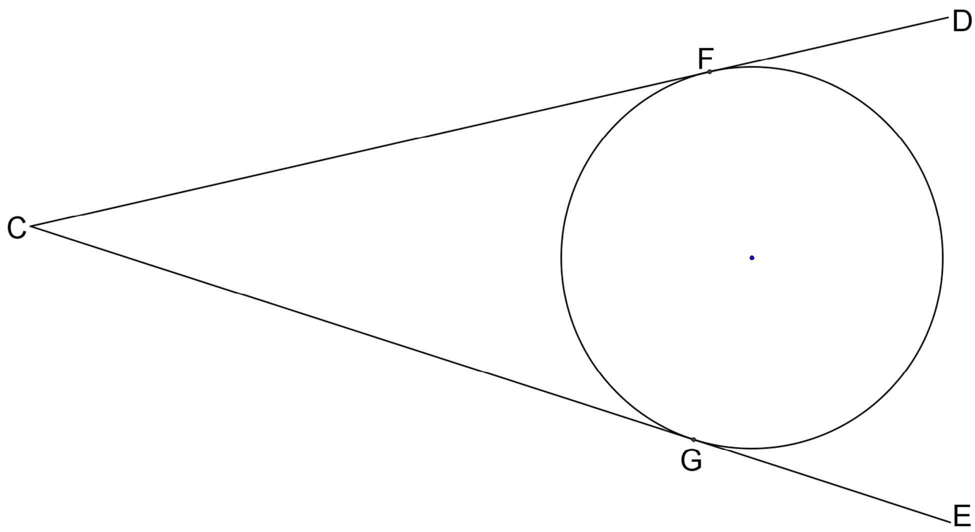


In the accompanying figure, GIH is tangent to the circle and points J, I, K, D, M, N and L are on the circumference of the circles.

Instructions	Conclusion
16. Shade the size of angle GIJ and search for an angle equal to that in the same circle # Repeat that for angle HIK. 17. Shade the size of angle MDN and search for an angle equal to that in the same circle. # Repeat that for angle DML.	

<u>Theorem 16:</u>

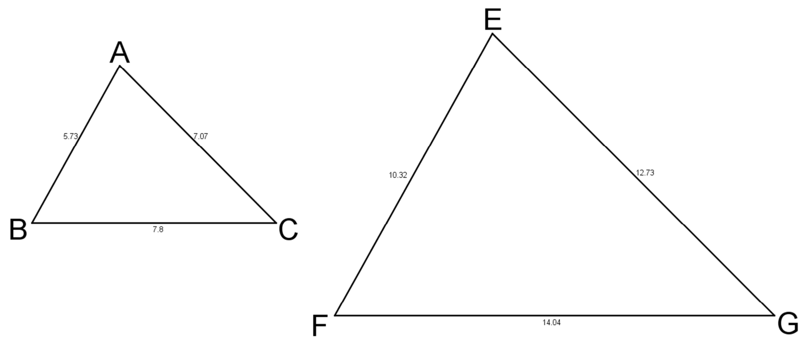
Theorem 17:



In the accompanying circle, CD and CE are tangents of the circle at F and G respectively.

Instructions	Conclusion
18. Measure the lengths of line segments CF and CG.	

Theorem 18:



Instructions	Conclusion
19. Measure Â in ΔABC and search for an angle in ΔDEF equal to it. Repeat #19 for B̂ and Ĉ. Measure sides opposite equal angles in the two triangles and find the ratios of those corresponding sides.	

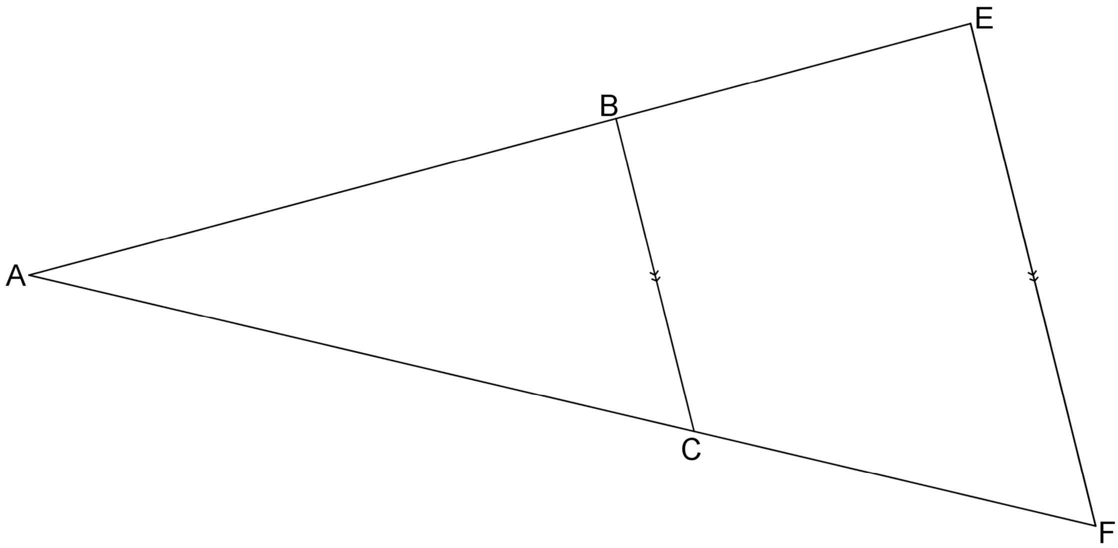
NOTE:

- When two triangles have corresponding angles equal, then they are said to be **similar.** Symbolically, it represented as $\Delta ABC/// \Delta DFE$
- When ratios of corresponding sides are equal (which is also referred to as corresponding sides being proportional), the two triangles are **similar.**
- Sides of two triangles being proportional imply that corresponding angles are equal and vice versa.

Theorem 19:

Congruency vs Similarity

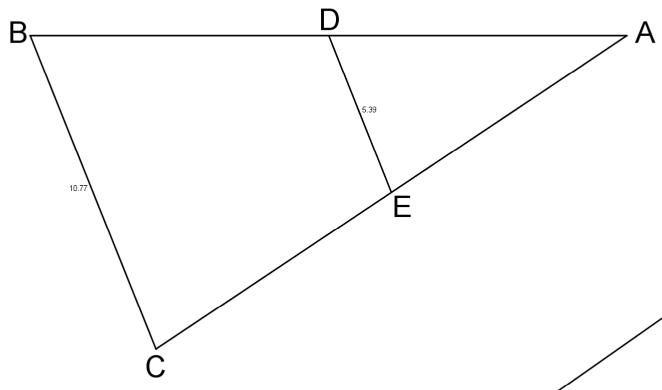
- Any two or more triangles which are congruent are also similar, but if two triangles are similar, they may not necessarily be congruent.

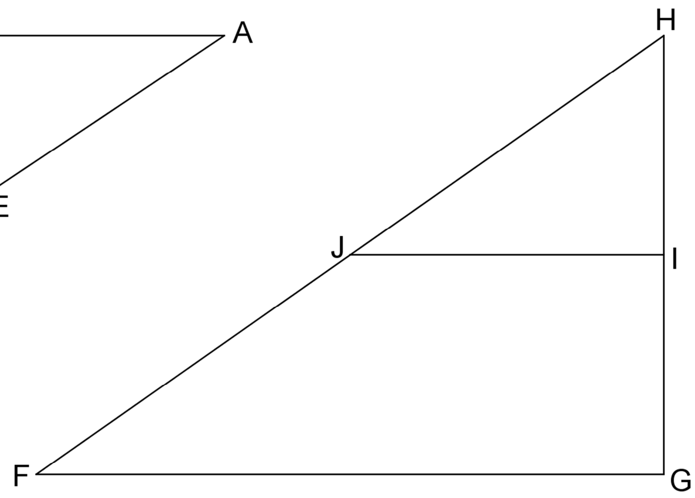


Verify that BC // EF by measuring (shading) and comparing corresponding angles.

Instructions	Conclusion
20. Measure the lengths of line segments AB and BE and calculate the ratio AB : BE or $\frac{AB}{BE}$ and repeat that for AC and CF. Check what happens using your measurements to compare the manipulations such as $\frac{AB+BE}{BE}$ or $\frac{AB}{AB+BE}$ done on both sides of your conclusion.	

<u>Theorem 20:</u>

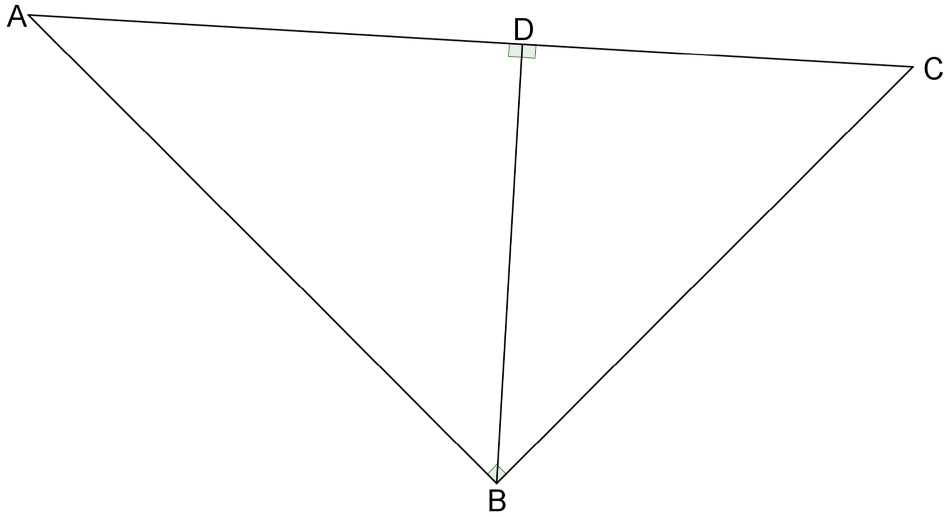




Verify that $AD = DB$ and $AE = EC$ in ΔABC and $FJ = JH$ and $GI = IH$ in ΔFGH.

Instructions	Conclusion
20.1. Measure the lengths of line segments DE and BC in ΔABC and compare the two. # Do the same for JI and FG in ΔFGH. Shade the size of AD̂E in ΔABC and look for an angle equal to that. # Repeat that for HĜF in ΔFGH. What conclusions can you make from the findings above on line segments DE and BC of ΔABC and JI and FG of ΔFGH?	

Theorem 20.1:



Verify if the angles marked are indeed 90°

Line segment BD is an   of ΔABC drawn from the right angle.

Instructions	Conclusion
20.2. Shade the size of $\angle A$ in ΔABC and look for an angle equal to that anywhere on the diagram. Repeat that for $\angle C$. Then the three triangles are *What conclusions can you make from the findings above about the sides of triangles ΔABC, ΔADB and ΔBDC?	

Theorem 20.2:

Instructions	Conclusion
20.3. Choose amongst the conclusions in * above that involve ΔABC and pick the fractions that have a common line segment in the two equal fractions. Cross multiply each equation Add the two equations and factorize and simplify where applicable Which popular theorem have you proved	

Warm-Up Activity:

1. Determine the areas of the shaded parts.
2. Without any further measurements, show that $\angle ABC = 90°$.