Factorisation in Heavy Ion Collisions

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Abstract. We present our latest findings on the status of factorisation in heavy ion collisions. We show that energy loss calculations that assume factorisation yield results consistent with factorisation: the leading order in energy asymptotics for the mean transverse momentum squared picked up by a high energy particle propagating through a quark-gluon plasma is double logarithmic. Further, the leading order behaviour for the difference in jet sizes in medium vs. in vacuum is negative; i.e. we predict jet narrowing in heavy ion collisions. This qualitative result is consistent with recent experimental measurements.

1. Introduction

A microsecond after the Big Bang the universe cooled to a chilly trillion degrees, at which point all of space was filled with a novel state of matter: the quark-gluon plasma (QGP). As the universe continued to cool and expand, the dynamics of this early stage of its history imprinted itself; further dynamics then propagated these inital conditions to the large-scale structure of the universe as we know it today.

From a theoretical perspective, the non-trivial, emergent many-body dynamics of quantum field theories is an active and interesting open area of research [1–4]. Even "simple" systems that depend only on the Abelian electromagnetic force show a wealth of extremely important behaviours that are currently not well understood from first principles: e.g. the phase structure of water [5] or high temperature superconductivity [6]. We're naturally led to consider the non-Abelian generalisation of many-body dynamics in quantum field theories in order to compare and contrast with the Abelian case and also because the non-Abelian case may be in some ways richer and in some ways simpler than the Abelian one [1–4, 7].

Experimentally, incredibly, we have the ability to probe these non-trivial, emergent, many-body dynamics of a non-Abelian theory and also the physics of the early universe through heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC). One of the most important experimental tools for investigating the properties of the QGP produced in these heavy ion collisions is known as "hard probes" [8]. Hard probes are particles with a large scale (mass or energy) that are produced in the initial overlap of nuclei in these nuclear collisions. These particles subsequently propagate through the medium created by the collisions. The idea is that measuring the difference in distribution of these particles in heavy ion collisions compared to the distribution of these particles when produced in much smaller collision systems (in angle, momentum, etc.) will provide insight into the properties of the QGP.

In order to connect the measured distribution of particles to properties of the QGP, we need theoretical calculations. For the particular case of hard probes, one avenue is to assume that the hard probes is weakly coupled to a weakly coupled QGP medium. One then derives expressions for the amount of energy lost from the hard probe to the QGP as the probe propagates through the medium [9–17]. Phenomenological models built on these energy loss calculations have shown great success in describing experimental data [18–20]. One important avenue for research going forwards is to put these energy loss derivations on more solid theoretical footing. In particular, it's important to understand how the corrections to the current leading order results might scale with, e.g., the energy of the probe.

2. Factorisation in QCD

In many simpler QCD systems, e.g. deep inelastic scattering (DIS), semi-inclusive deep inelastic scattering (SIDIS), Drell-Yan production (DY), etc., theoretical predictions are known to be of a *factorised* form [21]. These factorised formulae share two important aspects. First, the high-energy (or hard scale), short distance physics is factorised from the low-energy, non-perturbative physics. Second, it's known that the corrections to these factorised formulae are down by a very large energy scale $\sim 1/Q^2$, $Q \gg \Lambda_{QCD}$, where Λ_{QCD} characterises the energy scale at which non-perturbative physics sets in in QCD. What we would like to do, then, is work towards a factorised form for energy loss calculations in heavy ion collisions.

As a first step in that programme, we would like to compare a result computed within the factorised approach and one within the energy loss approach. One such observable is the mean transverse momentum squared picked up by the hard probe as it propagates through the medium, $\langle p_T^2 \rangle$. The factorised approach to this SIDIS-type calculation has been computed to next-to-leading order accuracy [22, 23], which is to say up to corrections including radiative (energy loss) emissions. Similar to other factorised DIS and SIDIS calculations [21], this factorised approach yields a type of parton distribution function with DGLAP-like evolution equations induced by the NLO contributions. Since the leading order contribution is from elastic scattering and should grow like $\log(E)$, we expect that the evolution equations will lead to an additional logarithmic growth in energy. While an explicit calculation has not been performed yet, we thus expect an overall $\log^2(E)$ dependence from the factorised approach for $\Delta\langle p_T^2 \rangle$, the *difference* in transverse momentum squared picked up by the parton in medium minus the transverse momentum picked up by the parton through vacuum radiation emissions. In the following, we investigate the leading energy asymptotics of $\langle p_T^2 \rangle$ as computed within the energy loss approach.

3. Energy Loss at High Energy

In the limit of massless particles and the soft and collinear emission of gluon radiation off of a high-energy parton propagating through a weakly-coupled QGP, the single inclusive distribution of emitted radiation in medium (minus the radiation emitted by a hard scattering in vacuum) is given by [11]:

$$\frac{dN^g}{dx d^2\mathbf{k}_T d^2\mathbf{q}_T} = \frac{C_R \alpha_s}{2\pi^2} \frac{L}{\lambda} \frac{1}{k_T^2} \frac{\mu^2}{\pi(q_T^2+\mu^2)^2} \frac{2\mathbf{k}_T \cdot \mathbf{q}_T (\mathbf{k}_T - \mathbf{q}_T)^2}{(4xE/L)^2 + (\mathbf{k}_T - \mathbf{q}_T)^4}, \tag{1}$$

where C_R is the colour Casimir relevant for the gluon or quark parton, $\mathbf{q}_T$ is the transverse momentum picked up by the parton from the medium, x is the (lightcone plus) momentum fraction taken by the emitted gluon from the parton, $\mathbf{k}_T$ is the transverse momentum of the emitted gluon, $L \sim 5$ fm is the length of the plasma traversed by the parton, $\lambda \sim 1$ fm is the mean free path of a gluon in the plasma, and $\mu \sim 0.5$ GeV is the Debye mass of the plasma. The upper bound of the $\mathbf{q}_T$ integration is set to $q_{max} \equiv \sqrt{3\mu E}$, which is the kinematic bound for elastic $2 \to 2$ scattering of two massless particles, one with energy E and one with a thermal momentum of $\sim 3\mu$. The upper bound of the $\mathbf{k}_T$ integration is $k_{max} \equiv 2x(1-x)E$, which ensures that the emission of the radiation is approximately collinear.

Note that because equation (1) is a *difference* in radiation distributions, there are regions of phase space for which $dN^g/dxd^2\mathbf{k}_T d^2\mathbf{q}_T < 0$, indicating the importance of quantum mechanical destructive interference in the QGP case: the presence of the QGP medium *suppresses* the emission of radiation in some cases, leading to less overall radiation than when the medium is not present.

Since equation (1) is a single inclusive distribution, the number of emitted gluons is not fixed (CITE multigluon). (For the typical values of μ, L, and λ quoted above, the total number of emitted gluons is ~ 3.) Thus when we compute the $\Delta\langle p_T^2\rangle$ of the emitted parton, we should simply compute:

$$\Delta\langle p_T^2\rangle \equiv \langle p_T^2\rangle_{QGP} - \langle p_T^2\rangle_{vacuum} = \int dxd^2\mathbf{k}_T d^2\mathbf{q}_T (\mathbf{k}_T - \mathbf{q}_T)^2 \frac{dN^g}{dxd^2\mathbf{k}_T d^2\mathbf{q}_T}. \tag{2}$$

4. Asymptotic Analysis

Numerical evaluation of equation (2) is difficult. While the integral converges, the integral only barely converges. The reason the integral only barely converges is that the integrand is composed of several terms. If the separate terms are integrated individually, they diverge. Only when the terms are integrated together are there the correct, delicate cancellations needed for the total integral to converge. It's generally difficult for numerical integration routines to fully capture such a delicate cancellation. An analytic handle on the result is therefore desirable. One approach to approximating equation (2) is to perform a change of variables to $\mathbf{q}' \equiv \mathbf{k}_T - \mathbf{q}_T$. This shift in integration variables significantly simplifies the integrand at the cost of complicating the integration region. The integral is broken up into three regions:

$$\begin{aligned}\Delta\langle p_T^2\rangle = &\int_0^{x_{min}} dx \int_0^{k_{max}(x)} d^2\mathbf{k}_T \int_0^{q^+_{max}(\mathbf{k}_T)} d^2\mathbf{q}' I \\ &+ \int_{x_{min}}^1 dx \int_0^{q_{max}} d^2\mathbf{k}_T \int_0^{q^+_{max}(\mathbf{k}_T)} d^2\mathbf{q}' I \\ &+ \int_{x_{min}}^1 dx \int_{q_{max}}^{k_{max}(x)} d^2\mathbf{k}_T \int_{q^-_{max}(\mathbf{k}_T)}^{q^+_{max}(\mathbf{k}_T)} d^2\mathbf{q}' I,\end{aligned} \tag{3}$$

where the integrand I is the same in all three regions and is given by:

$$I \equiv \frac{C_R\alpha_s}{2\pi^2}\frac{L}{\lambda}\frac{1}{k_T^2}\frac{\mu^2}{\pi\big((\mathbf{k}_T-\mathbf{q}')^2+\mu^2\big)^2}\frac{2\mathbf{k}_T\cdot(\mathbf{k}_T-\mathbf{q}')q'^4}{(4xE/L)^2+q'^4}. \tag{4}$$

The $\mathbf{q}'$ integration limits are given by:

$$q^\pm_{max} \equiv k_T\cos(\theta_{kq}) \pm \sqrt{(q_{max})^2 - k_T^2\sin^2(\theta_{kq})}, \tag{5}$$

where θ_{kq} is the angle between the $\mathbf{k}_T$ and $\mathbf{q}'$ vectors, and $x_{min} \equiv \sqrt{3\mu/4E}$.

In order to make our job of analysing equation (2) easier, we will take $k_{max} = 2xE$. Since spin-1 radiative emissions are dominated by small x, we expect this to be a good approximation.

Numerical investigation of the three regions shows that the first two contributions grow with $\log^2(E)$ while the third region grows only with $\log(E)$. The overall $\log^2(E)$ growth is reassuring as it should match what we believe will be the leading double logarithmic energy dependence from the factorised approach as noted above.

To derive analytic expressions from the first t wo i ntegrals o f e quation (3), i t's u seful to approximate the upper bound on the $\mathbf{q}'$ integrals as infinity. Numerical investigation shows that this approximation makes little difference in the overall results, especially as one increases in energy. Intuitively, one can understand this insensitivity as follows: the dominant contribution to the integrals comes when $\mathbf{k} \sim \mathbf{q}'$, since these values minimise the denominator in equation (4). Physically, the greatest transverse momentum transfer to the parton from the radiation occurs when the parton has the smallest momentum transfer from the medium.

Once the $\mathbf{k}$ dependence is gone from the $\mathbf{q}$ integration, we may readily analytically integrate equation (4) over $k_T = |\mathbf{k}|$ and θ_{kq}. (A trivial 2π falls out of the extra angular integral, as per usual.) The integration over x is less easy, but still analytically tractable; the expression is long and not insightful. Clever rearrangement of terms leads to analytically tractable integrands that numerical investigation show grow with energy and terms that do not. The final r esult, correct to leading logarithms in energy is:

$$\Delta\langle p_T^2\rangle = -\frac{C_R\alpha_s}{4}\frac{L}{\lambda}\mu^2\left[\log^2\left(\frac{4E}{\mu^2 L}\right) + \frac{5\pi^2}{12}\right]. \tag{6}$$

5. Conclusions

One can see from equation (6) that the leading double logarithmic term for the change in the mean transverse momentum squared picked up by a parton emerging from nuclear collisions is negative, which is to say that jets are *narrowed* by the presence of the QGP medium. Further, the effect scales as one might expect. Higher energy jets are narrowed more by the QGP than less high energy jets, and the narrowing increases with increasing pathlength and Debye screening and for gluon vs. quark jets; i.e. the greater the quantum interference—from a greater amount of induced radiation—the more the jet is narrowed. The leading double logarithmic energy dependence is also what one expects if factorisation should hold for the system: elastic energy loss leads to a logarithmic dependence on energy and DGLAP-like evolution will add an additional log.

This qualitative prediction of the narrowing of jets is consistent with preliminary results from the ALICE Collaboration at the LHC [24].

Interesting further work includes quantifying the differences between the energy loss approach and the factorisation approach, and considering higher order effects such as from small path lengths [25], small system sizes [26, 27], or the flow of the medium [28].

Acknowledgements

The author gratefully acknowledges generous support from the South African National Research Foundation and the SA-CERN Collaboration. The author further gratefully acknowledges extensive valuable discussions with Matthew D. Sievert and Hannah Clayton.

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