Statistical correlations impacting a top quark mass measurement in 13 TeV proton-proton collision data from the ATLAS detector

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Abstract. The top quark is the heaviest particle in the Standard Model and reducing the uncertainty of the top quark mass directly speaks to/ affects precision tests of the consistency with the Standard Model, where breaks from this consistency would point to the existence of more massive particles beyond the Standard Model. Since the top quark decays before hadronizing, either the kinematic properties of the decay products or measurements of the rate of the top quark production have been used to measure the mass of the top quark. The majority of measurements consider various decay modes of the W boson with no specification on the decay of the b-quark when utilizing the kinematic properties of the decay products. These measurements are predominantly limited by uncertainties related to the reconstruction of jets. However, there is a top quark decay mode which is largely independent of the aforementioned uncertainty which requires large amounts of data due to its low production rate. This decay mode includes a J/ψ meson originating from a b-hadron and a semi-leptonic decay of the W boson. The invariant mass reconstructed from the J/ψ meson and lepton is sensitive to the top quark mass. This paper describes a maximum likelihood approach to extract the top quark mass from a probability density function, pdf, while studying the impact of the correlations between each of the pdf parameters.

1. Introduction

The top quark is one of the most interesting fundamental particles in the Standard Model (SM) because the top mass impacts many areas within the SM and in new physics Beyond the SM (BSM) [1–3]. The current average Monte Carlo (MC) top mass of 172.26 $\pm$ 0.30 GeV [4] was determined by combining the kinematics of the top quark's decay products, i.e. jets [5] and leptons. These jets and leptons originated from a primary decay of the top quark (i.e. into a W boson and a b quark) which has a branching fraction of 95.7$\pm$3.4% [4]. There are other ways to measure the top mass which takes advantage of top quark cross section measurements [2], as well as different ways to quote the top mass depending on the renormalization scheme [6].

The largest source of uncertainty associated to top mass measurements using jet kinematics is jet reconstruction [7–10]. There are however different top quark decay modes that depend less on jet reconstruction and may produce a more precise top mass measurement. The decay mode of interest consists of a lepton originating from a W boson and two oppositely charged muons decaying from a J/ψ meson which originated from the b quark. This decay mode is largely independent of the jet reconstruction uncertainty. Due to the large amount of data recorded by the Large Hadron Collider (LHC) during 2015-2018 [11], this decay mode can now be probed [12].

The invariant mass reconstructed from the lepton from the W boson and two oppositely charged muons from the J/ψ (i.e. three lepton system) is sensitive to the top mass [13]. This paper describes a maximum likelihood approach to extract the top quark mass from a probability density function (pdf) while studying the impact of the correlations between each of the pdf parameters using simulated proton-proton collision data at $\sqrt{s} = 13$ TeV, corresponding to an integrated luminosity of 139 fb^{-1} [14, 15].

2. ATLAS Detector

ATLAS [16] is a general-purpose detector designed to capitalize on the full potential of the LHC [11]. The magnet system consists of a superconducting solenoid surrounding the Inner Detector (ID) and three large superconducting toroids, one barrel and two end-caps, arranged azimuthally symmetric outside the calorimeters and within the Muon Spectrometer (MS). The ID performs precise particle reconstruction and identification of the collision point over $|\eta| < 2.5$, while the calorimeters measure the energy and position of particles over $|\eta| < 4.9$. The MS surrounds the calorimeters and identifies and measures muon up to $|\eta| < 2.7$. These components are integrated with a Trigger and Data Acquisition system and a computing system which selects events which consist of high transverse momenta particles or large missing transverse energy, and stores them for further analysis.

3. Event Selection

The ℓ+J/ψ top quark decay signature requires the W boson to decay leptonically into either an electron or a muon, at least 1 b-tagged jet and two oppositely charged muons with an invariant mass around that of a J/ψ meson (i.e. 3096.900 ± 0.006 MeV). To enhance the selection of a lepton originating from a real W boson, a quantity known as the transverse mass of the W boson, $\mathrm{m_T}(\mathrm{lepton},\mathrm{E_T^{miss}}) = \sqrt{2p_\mathrm{T}(\mathrm{lepton})\mathrm{E_T^{miss}}(1 - \cos(\phi(\mathrm{lepton}) - \phi(\mathrm{E_T^{miss}})))}$, was used. This quantity combines the kinematics of the lepton and the missing transverse energy (which represents the transverse kinematics of the neutrino). The kinematic requirements to extract the ℓ+J/ψ top quark decay signature is summarized in Table 1, where the definitions of the different observables can be found in reference [17–20]. The background estimation of this event selection can be found in [21].

Table 1. A table with the kinematic criteria for t $\rightarrow \ell$+J/ψ events is shown.

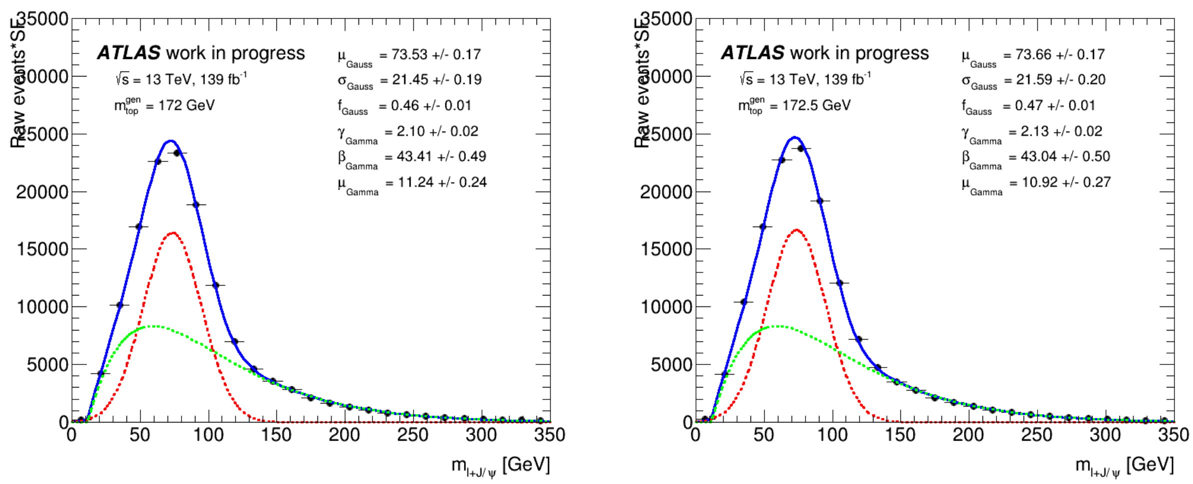
Description	Kinematic criteria
Electron (Muon) selection	Exactly 1 lepton with $\mathrm{p_T} > 25$ GeV, $|\eta| < 2.5$ $|\Delta z_0 \sin\theta| < 0.5$ mm, $|d_0|/\sigma_{d_0} < 5$ (3), isolation Gradient
Jet selection	At least 1 b-tagged jets with $\mathrm{p_T} > 25$ GeV & $|\eta| < 2.5$
Missing transverse momentum selection	$\mathrm{E_T^{miss}} > 20$ GeV
Real W boson selection	$\mathrm{m_T}(\mathrm{lepton},\mathrm{E_T^{miss}}) > 40$ GeV
J/ψ muon selection	$\mathrm{p_T} > 2.5$ GeV if $|\eta| < 1.3$, $\mathrm{p_T} > 3.5$ GeV if $1.3 < |\eta| < 2.5$
J/ψ selection	$\mathrm{p_T} > 8$ GeV, $|y| < 2.1$, $2.9 < \mathrm{m}(\mu^+\mu^-) < 3.3$ GeV, $\tau > 0$ ps

4. Building the model to extract the top mass

The top mass can extracted using a maximum likelihood method with the invariant mass of the three-lepton system. Several top quark pair events were simulated within ATLAS using different top quark masses (i.e. 169, 171, 172, 172.25, 172.5, 172.75, 173, 174 and 176 GeV). The three-lepton mass distribution can be modelled by the sum of a Gaussian [22], representing the signal component, and a Gamma distribution [22], representing the background component. The signal component originates from pairing the lepton from the W and the two-oppositely charged muons from the J/ψ from the same top quark, whereas the background component originates from pairing the leptons from different top quarks as well as from possible non-correlated backgrounds [13].

4.1. Parameter correlations

The parameter values in the pdf can be determined by fitting over the three-lepton mass distribution from the different top masses. Figure 1 shows the data distribution with the pdf fit over the $m_{top} = 172$ GeV (a) and $m_{top} = 172.5$ GeV (b) $t\bar{t}$ MC samples. The y-axis shows the number of raw events while accounting for the different detector effects. The relationship of these parameters to the top mass can then be determined to form a pdf that is solely dependent on the top mass. However, these parameters are correlated to some degree (see Table 2) and could affect the model.



(a) (b)

Figure 1. The three-lepton mass distribution showing the pdf fit (blue) with the signal (red) and background (green) over the $m_{top} = 172$ GeV (a) and $m_{top} = 172.5$ GeV (b) $t\bar{t}$ MC samples. The y-axis shows the number of raw events while accounting for the different detector effects.

Determining the relationships between each parameter and the top mass can be done by fitting a straight line. However, this does not account for the correlations between the different parameters. A χ^2 function that incorporates the different correlations by using the inverse of the covariance matrix as discussed in G. Cowan - Statistical data analysis [23] and is shown below

$$\chi^2(\boldsymbol{\theta}) = \sum_{i,j=1}^{N} (y_i - \lambda(x_i;\boldsymbol{\theta}))(V^{-1})_{ij}(y_j - \lambda(x_j;\boldsymbol{\theta})) \tag{1}$$

Table 2. A table showing the correlation strengths between the pdf parameters after fitting over the m$_{top}$ = 172.5 GeV $t\bar{t}$ MC sample.

Parameters	μ_{Gauss}	σ_{Gauss}	fraction$_{Gauss}$	γ_{Gamma}	β_{Gamma}	μ_{Gamma}
μ_{Gauss}	1	-0.37	-0.37	-0.10	-0.11	-0.23
σ_{Gauss}	-0.37	1	0.83	-0.41	0.58	0.63
fraction$_{Gauss}$	-0.37	0.83	1	-0.52	0.73	0.57
γ_{Gamma}	-0.10	-0.41	-0.52	1	-0.92	-0.72
β_{Gamma}	-0.11	0.58	0.73	-0.91	1	0.66
μ_{Gamma}	-0.23	0.63	0.57	-0.72	0.66	1

where y_i are the measured parameter values, $\lambda(x_i;\boldsymbol{\theta})$ are the expected values (or straight line equations), and V_{ij}^{-1} is the inverse of the covariance matrix. The straight line equations take the form parameter = a(m$_{top}$-172.5)+b. The various values for a and b can be found in Table 3 for the different parameters with and without accounting for the correlations. Without considering the correlations, all the parameters except the fraction of Gaussian shows top mass dependence. The Gamma's parameter's were not expected to have any top mass dependency. However, including the correlations removed the top mass dependence in the Gamma's parameters, and only the Gaussian's parameters show a top mass dependence.

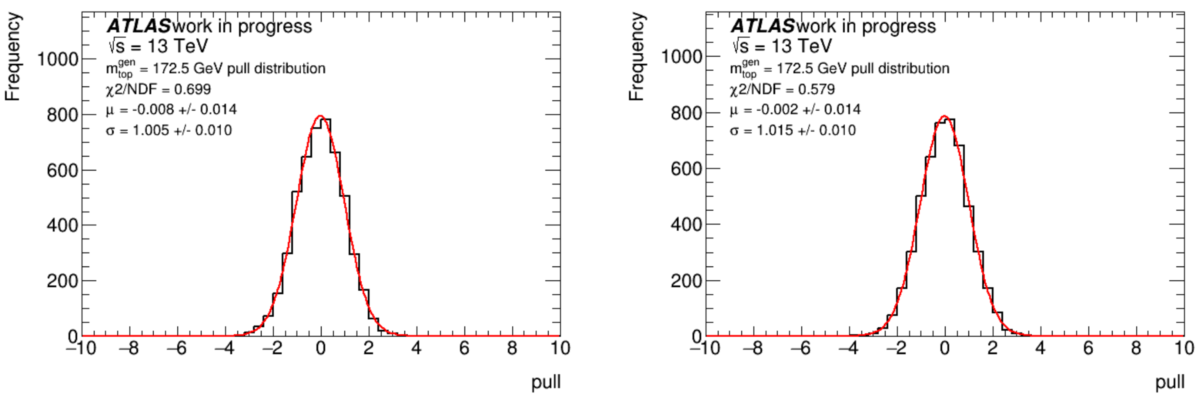
Table 3. A table showing the parameter relationships as a function of the top mass with and without accounting for the parameter correlations.

Parameter	Without correlations	With correlations
μ_{Gauss}	a = 0.502 ± 0.031	a = 0.538 ± 0.028
	b = 73.765 ± 0.060	b = 73.775 ± 0.059
σ_{Gauss}	a = 0.187 ± 0.030	a = 0.171 ± 0.024
	b = 21.447 ± 0.059	b = 21.437 ± 0.058
fraction$_{Gauss}$	a = 0.00002 ± 0.00002	a = -0.00003 ± 0.00002
	b = 0.462 ± 0.002	b = 0.459 ± 0.003
γ_{Gamma}	a = 0.004 ± 0.004	a = -0.0001 ± 0.0001
	b = 2.111 ± 0.007	b = 2.101 ± 0.009
β_{Gamma}	a = 0.1660 ± 0.0784	a = -0.0015 ± 0.0016
	b = 43.613 ± 0.160	b = 43.542 ± 0.208
μ_{Gamma}	a = -0.065 ± 0.028	a = 0.0014 ± 0.007
	b = 9.691 ± 0.062	b = 9.789 ± 0.078

4.2. Testing the performance of the model

The parameter relationships can be substituted into the pdf yielding a pdf with only the top mass as a parameter. The pdf will then determine the top mass which best fits the data distribution. However, the performance of the model has to be tested. This was done by performing a pull study which tests the bias and error coverage of the fit. The pull is defined by the measured value minus the true value divided by the error in the measured value and is Gaussian in shape. The pull mean describes the bias in the measured value while the pull width describes the accuracy in the error. For no bias and an accurate error estimation, the pull mean and width must be consistent with 0 and 1 within standard deviation (or 1σ), respectively.

In order to create a pull distribution, 5000 pseudo-datasets were randomly generated by inserting the true mass value into the pdf and fitting the pdf over each pseudo-data to obtain the measured value and error of the top mass. Figure 2 shows the pull distributions without (a) and with (b) accounting for the correlations when the true mass values is $m_{top} = 172.5$ GeV. A Gaussian was fit over the pull distribution to determine the pull mean and width. This was done for the nine different top mass values to estimate the pull mean and width as a function of the top mass.



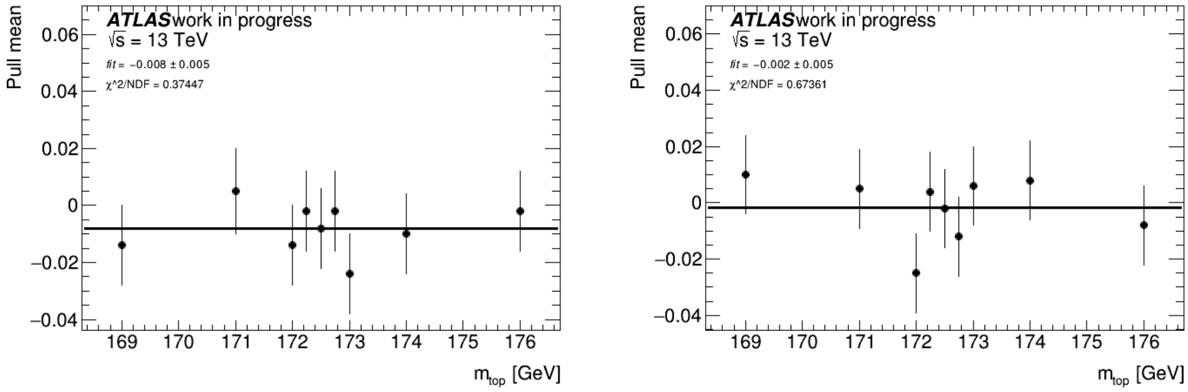
(a) (b)

Figure 2. The pull distribution without (a) and with (b) accounting for the various parameter correlations in the pdf when using $m_{top} = 172.5$ GeV as the true top mass value. The red line shows a Gaussian fit over the distribution.

Figure 3 shows the pull mean without (a) and with (b) accounting for the parameter correlations as a function of the top mass. The expected pull mean of the model is not consistent with 0 within 1σ when not considering, but is consistent with 0 within 1σ when considering the parameter correlations. This shows that without considering the parameter correlations, the model would have produced a bias in the measured value. Figure 4 shows the pull width without (a) and with (b) accounting for the parameter correlations as a function of the top mass. The expected pull width of the model is not consistent with 1 within 1σ when not considering, but is consistent with 1 within 1σ when considering the parameter correlations. This shows that without considering the parameter correlations, the model would have underestimated the error in the measured value.

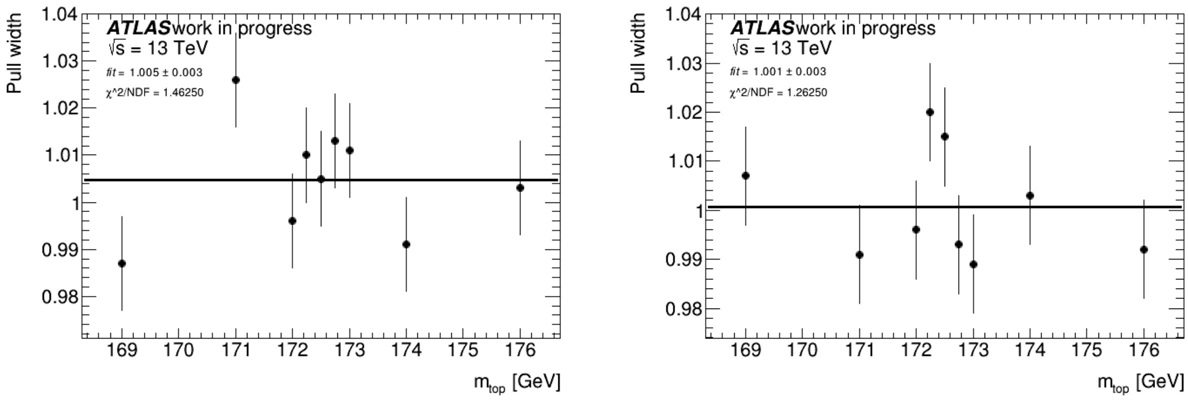
5. Conclusion

Measuring the MC mass of the top quark using the $\ell + J/\psi$ decay signature reduces the dependency on jet reconstruction. The three-lepton mass distribution is sensitive to the top mass which can be determined using a maximum likelihood approach. This maximum likelihood approach contains a pdf that consists of a Gaussian (signal) and a Gamma function (background). The parameters from the Gaussian and Gamma functions are correlated in the pdf. Not accounting for these statistical correlations in the parameters would have produced a bias and underestimated the uncertainty in the top mass measurement in data. After accounting for these parameter correlations, the model showed to have no bias and accurately estimates the uncertainty in the top mass measurement.



(a) (b)

Figure 3. The pull mean as a function of the top mass without (a) and with (b) accounting for the parameter correlations. The solid black line represents the expected pull mean of the model and the value can be found on the top left of the figure.



(a) (b)

Figure 4. The pull width as a function of the top mass without (a) and with (b) accounting for the parameter correlations. The solid black line represents the expected pull width of the model and the value can be found on the top left of the figure.

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7. References

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